HEAT

An Elementary Text Book

IVOR B. HART

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AN ELEMENTARY TEXT-BOOK

BY

IVOR B. HART

O.B.E., PH.D., B.SC.

AUTHOR OF "AN INTRODUCTION TO ADVANCED HEAT"
"MAKERS OF SCIENCE" ETC.

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PREFACE

In this work the whole of the elementary portions of the author's *A Student's Heat*—published some years ago, and now out of print—have been thoroughly revised. Accordingly, all the acknowledgments therein made apply equally to this book. Great pains have been taken to eliminate errors, and the text has been altered and added to in many respects. Throughout the book worked examples have been freely inserted, and both these and the questions set at the ends of the chapters have been made of as practical a character as possible.

It is an important fact upon which insufficient stress is laid in most school text-books on heat that whilst the C.G.S. system of units must necessarily remain fundamental in science, in the everyday experiences of life in this country the F.P.S. system is the one most usually met. Accordingly this system, together with the Fahrenheit temperature scale, is given sufficient prominence in both text and examples, so that it is hoped that students will be equally competent in the handling of either system.

The standard of the book is approximately that required for University Matriculation and its various equivalents. The author is indebted to Messrs. Philip Harris & Co. Ltd. for the use of a number of blocks, and to Messrs. J. R. Smith, B.Sc., and W. Laidler, B.Sc., for assistance in the working out of answers to problems and in the reading of proofs.

IVOR B. HART

April 1925

NOTE TO FOURTH IMPRESSION

In this reprint the opportunity has been taken to improve a number of the diagrams, and to correct a few errors. In this connection the author desires to thank those teachers who have communicated with him.

IVOR B. HART

January 1930

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HEAT

CHAPTER I

THE MEASUREMENT OF TEMPERATURE

Introduction.—It is a curious fact that whereas science in general has a history which carries us back to Greek times, the study of heat in particular is comparatively recent. Warmth has always been necessary to mankind, and so some ideas were always present on the subject; but they were hardly scientific. The best-developed branch of physics in early days was that of "optics," and by the seventeenth century this study was in a distinctly advanced state. Yet the serious study of heat was then only beginning. In the days of Greek culture, the idea was put forward that the ingredients of the world were earth, air, fire, and water, and that these, taken two at a time in combination with the four primary *qualities* of hot and cold, wet and dry, could make up all the known forms and conditions of matter. Heat and cold, therefore, were distinct, different, and opposite. Of course this could explain many existing facts and phenomena, and so the notion was satisfying. So much so, indeed, that it persisted for many centuries.

Like so many other branches of science, little real headway could be made until the importance of experiment was appreciated. It is sometimes difficult for the modern schoolboy to realise, in these days of well-equipped laboratories, that for hundreds of years such a thing as an experiment was scarcely thought of. Before the sixteenth century practically all questions were settled by appeal to authority—to the writings of such great philosophers as *Aristotle* (385–323 B.C.), *Archimedes* (*circa* 287–212 B.C.), and

Ptolemy (*d.* A.D. 168). The reputation of their writings was sufficiently great to make them the final authority on all matters. Obviously, while such a state of affairs existed, there could be no real progress for science, and it was not until two great scientific "revolutionaries"—*Galileo* (1564–1642) in Italy and *William Gilbert* (1544–1603) in England—broke away from tradition and insisted on the independent test of experiment in all things that a new outlook was initiated and a new hope appeared for science.

The Theory of Caloric.—One of the immediate effects of experiment upon scientific inquiry was the invention of measuring instruments, and, with the inauguration of exact measurement, progress in science was inevitable and rapid. In heat, for example, the invention of the thermometer by Galileo in 1593 was the starting-point of a new science, which it is the purpose of this book to discuss. One of the firstfruits of the new attitude to the subject was a desire to inquire into the *nature* of heat. What is it? An answer to this question was provided by a development in the sister study of chemistry. One of the most famous of the seventeenth-century chemists was *George Ernest Stahl* (1660–1734), of the University of Halle. This philosopher taught that when a body burns it gives off a substance which he called **phlogiston.** Since burning and heat are associated ideas, there developed in consequence a corresponding view regarding heat. This was looked upon as a material substance called **caloric.** It was held to be highly elastic, and its particles were supposed to repel each other. When a body "gave out heat" it was also supposed to give out caloric particles, and the converse was held to be true for cooling.

We owe the downfall of the caloric theory of heat to *Count Rumford* (1753–1815), who in 1798 was engaged in cannon-boring operations at Munich, in Bavaria. Naturally he noticed the great heat produced during the grinding of the boring tool against the gun-barrel. "Whence," he asked, "came this heat?" The current theory said, "From the caloric that was being given out." But Rumford could find no sign of either the gun, or the chips, or the borer losing any weight. Again, if the boring could be continued long enough, the available stock of caloric ought to become exhausted. But there seemed no end to the heat he could

generate merely by going on boring. Did the heat then come from the air? Rumford repeated the experiment with the apparatus under water, and still the heat was produced. Rumford now definitely discarded the caloric theory. What, then, is heat?

The Molecular Theory of Heat.—Rumford now thought of some speculations that had been put forward, among others, by *Robert Boyle* in the latter half of the seventeenth century. Boyle (1627–1691) was one of the founders of the Royal Society, and was a contemporary of the illustrious *Sir Isaac Newton* (1642–1727). He advanced the view that **the small particles of which bodies are built up are in rapid, haphazard motion in all directions, and that this motion expresses itself as heat.** Cold is produced by a "less vehement agitation" of the particles, and any means which could be devised to increase this agitation would be found to produce heat. Boyle showed this by arranging two spherical pieces of brass, so made that the concavity of the one could be placed in intimate contact with the convexity of the other, to rotate in opposite directions inside an evacuated vessel. He tells us that the heat produced was so intense that he could not touch the brasses.

Thinking of this theory, Rumford concluded that the friction of the borer against the cannon would be sufficient to excite the particles (molecules, we now call them) to greater violence of motion, and therefore heat would be manifested. He then carried out his historic experiment of boring a brass cylinder with a blunt steel borer, the whole being immersed in water at a temperature of 60° F. After two hours and twenty minutes the water was boiling. This experiment was repeated in a modified form and under more exact conditions by *Sir Humphry Davy* (1778–1829) to meet the objection that the heat might yet have come from the *water* in Rumford's experiment. He arranged for two pieces of ice to rub against each other in a vacuum. The whole apparatus was kept at a temperature well below that of ice,[1] so that no heat could possibly pass to the ice from outside. Yet the ice melted.

Davy's conclusion has been accepted ever since, that **"friction causes vibration of the corpuscles of bodies, and this vibration is heat."**

[1] Freezing mixtures had been used as far back as the seventeenth century by Robert Boyle.

Sensation of Hotness.—Our first ideas of heat are derived from our sensations. As part of our nervous equipment, we possess certain nerves distributed all over the body. These tell us that a room is hot, or that the night is chilly, and so on. We become aware through their agency of different degrees of "hotness" or warmth, and we interpret these impressions by speaking of different **temperatures.** Thus we say that hot water is at a higher temperature than cold water. In the early days of physical science, the interpretations of temperature sensations were very vague, and comparisons of degrees of hotness or warmth amongst different observers was a matter of great difficulty. Nowadays, however, exact schemes of temperature gradations have been agreed upon, and the language of temperature is universal. The first step in this direction was the realisation of the unreliability of human sensations in interpreting temperature. A may be sitting in a room quietly reading, thinking meanwhile that it is a cold day, whilst B may presently enter after a 5-mile run and complain that the air is hot. Both cannot be right, and often both may be wrong; the true facts are so frequently obscured by the immediately past history of the human system.

The familiar experiment of the three bowls brings this out very well indeed. Take three bowls, A, B, and C; A containing cold water, B lukewarm water, and C hot water. Plunge the right hand in C and the left hand in A. After a minute transfer both hands into B. Then the left hand will feel hot in contrast with the coldness of A, and the right hand will feel cold in contrast with the hotness of C. Yet both are in the same bowl B, which is neither hot nor cold.

Our own sensations, therefore, are unreliable as indicators of temperature. Exact indications of temperature, however, form the basis of our study of heat, and it is very necessary to devise some means of ascertaining exactly the degree of hotness of a body. An instrument which does this is known as a **thermometer,** and as far as possible we endeavour to use in thermometry properties which bring our own sensations into play as little as possible. This "personal equation," as it is called, cannot entirely be avoided, and usually the nerves of vision have to be employed. These, however, are far more reliable than our "temperature nerves," and for most purposes the normal eyesight is adequate.

To avoid, then, the possibilities of error, we must take advantage of the effects which heat has upon inanimate objects. There are several such effects which are used in thermometry, the two chief being (1) the influence of heat upon the size of a body, and (2) the influence of heat upon the electrical properties of certain bodies. We shall develop the first of these two considerations in some detail.

The Influence of Heat upon the Linear Dimensions (*i.e.* Length, Area, or Volume) of a Body.—With a very few exceptions, which will be dealt with separately, all bodies expand on heating and contract on cooling. This is true for lines, areas, and volumes, and also for all three states of matter—solid, liquid, and gaseous. This property has, of course, been known for centuries, and we meet with examples of its action almost every day of our lives. The following simple experiment will serve to illustrate the effect in the case of solids and liquids. F is a flask (Fig. 1) filled with a liquid, and fitted with a cork through which passes a long vertical tube AB. The liquid completely fills the flask and part of the stem. Its level C is indicated by means of a piece of paper DE with graduations marked on it, fixed as shown. The whole is mounted on a tripod stand. Heat is now applied by means of a Bunsen burner, and the first effect noticed is that the level of the liquid in the stem falls a little. It then stops, and on further heating steadily rises. On allowing the apparatus to cool, the liquid once more falls in the stem, and reaches its original position at C on again attaining the temperature of the room. The explanation of the initial fall of the level of the liquid is as follows. On first applying the Bunsen flame, the heat affects only the glass. The heat does not instantaneously penetrate to the liquid within, and therefore the glass alone is first warmed. It expands, and thus compels some of the liquid in the stem to occupy the small increased volume in the flask, and the level in the stem falls. Soon, however, the heat reaches the liquid itself, which now expands, and is forced steadily up the stem. This ceases when

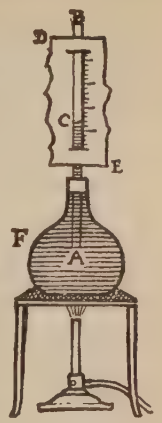

FIG. 1.

the flame is withdrawn, and the subsequent cooling produces the contraction which brings the liquid back to its original level.

The expansion of gases on heating is illustrated by the following experiment. Take a small flask, and gently warm it for some time over a Bunsen flame. Now invert it and place its open end in a beaker of water. It will be seen that the water rises up in the flask a considerable distance, the rise continuing until the flask has cooled to its original temperature. When it was first heated, the air within expanded, and much of it was forced out. On placing its open end under cool water, the flask was full of hot air. As this cooled, it contracted, and its pressure was therefore reduced below that of the atmospheric pressure exerted on the level of the water in the beaker. Equilibrium being thus disturbed, water was forced up the neck of the flask until the pressures inside and out once more became equal.

The Application of Expansion to Thermometry.—This property of the expansion of substances on heating at once affords a ready means of indicating the temperature of a body. If we know that the hotter a substance gets, the longer it gets, then by observing the variations in length which a body undergoes we are able to deduce the corresponding variations in temperature of which these differences in length are a consequence.

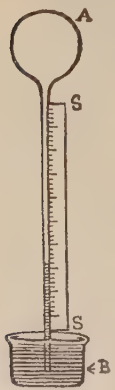

FIG. 2.

Hence the great importance of being able to measure exactly the amount of expansion which different substances undergo on heating. These measurements will be considered very fully in the next chapter. For the present, however, it will be sufficient to point out that in the case of solid bodies, the amount of expansion for ordinary ranges of temperature is too small to be employed usefully in thermometry. Thermometric measurement is therefore almost entirely confined to the expansion of liquids and gases.

The History of the Thermometer.—One of the earliest instruments on record was that used by Galileo in 1592 to illustrate his lectures. This consisted of a flask AB with a long narrow neck (Fig. 2), inverted with its end B dipping into a small reservoir of coloured water. Some

of the air within the flask having been withdrawn, the decreased pressure of the air so produced caused the water to rise in the neck. The position of the head of the column was indicated by a scale SS at the side. The effect of heat is much greater upon air than upon water, and hence, when the temperature rose, the liquid fell in the stem, and *vice versa*.

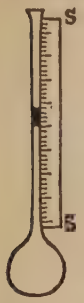

FIG. 3.

The chief error in this instrument arises from the fact that not only does the position of the liquid in the stem depend upon the temperature, but also upon the barometric pressure. Galileo, however, was probably unaware of this, as the barometer was not invented till about 1642.

Various modifications of this thermometer followed, until in 1603 it had taken the following form. The same flask was used, but was no longer inverted (Fig. 3). In place of the reservoir, a small column of coloured water was introduced into the neck of the flask, and kept in position by capillary attraction. It would rise and fall in the stem with the expansion and contraction of the air in the bulb, and the scale SS gave its exact position. This proved a very troublesome form, for apart from the barometric error, which of course is serious, the capillary attraction was often insufficient to keep the thread of liquid in position, and it would run down into the bulb.

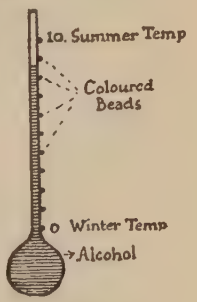

FIG. 4.

Further modifications became necessary. The Florentine Academy (a famous seventeenth-century school of physics) therefore definitely abandoned air as a thermometric substance, and used a flask having a small bulb and a long neck of fine bore. This was open at the top and was almost completely filled with liquid (Fig. 4). The barometric error was thus to some extent corrected, and with experience the makers were able so to proportion the instrument as to give a range of movement of the liquid in the stem from the bottom in winter to the top in summer.

In those days many philosophers were obsessed (as indeed many are to-day) with the idea of perpetual motion, and they

saw an example of this in the steady movement up and down of the liquid in the stem. Thus one writer speaks of the thermometer of his day as a "perpetuum mobile" showing degrees of hot and cold.

In 1641 for the first time, a tube was used which was closed at the top after the air had been driven out by heating, and the liquid used was alcohol.

Experiments now took a more widened range, and the need was strongly felt for a systematic method of graduation. The method adopted by the Florentine Academy was to take as the zero mark the indication of the liquid level when placed in a certain cellar in Florence in winter. The indication in summer gave what was known as the ten of the scale. These two were separated by ten equal divisions, the graduations taking the form of coloured beads (Fig. 4). It was soon found that the cellar temperatures differed in successive years, and it became necessary to adopt standard temperatures which could be relied upon to remain constant. All sorts of suggestions were made. One was to take the temperature of air during a frost as the lower fixed point, and that of melting butter as the upper fixed point. It was not till the eighteenth century that the temperatures of melting ice and boiling water were finally adopted.

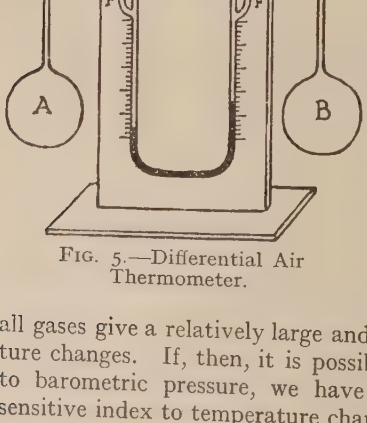

FIG. 5.—Differential Air Thermometer.

Air Thermometers. — It will be advisable, before we consider in detail the various kinds of liquid thermometers, to refer to air as a thermometric substance. We have seen that this was the first form of thermometer used, and although the variations in the atmospheric pressure made Galileo's thermometer unreliable, yet air and, indeed, all gases give a relatively large and uniform response to temperature changes. If, then, it is possible to overcome the errors due to barometric pressure, we have in air thermometers a very sensitive index to temperature changes.

A simple and useful form is that known as the differential air thermometer. This consists of two bulbs connected by tubing in the manner indicated in Fig. 5. By opening the stop-cock E in the horizontal connecting-tube CD the pressures on each side of the tube are equalised, and the liquid (introduced by FF) will therefore be at the same level as indicated by the scales on the stand. The stop-cock E is then closed. If now the bulb A is at a higher temperature than B, the air within expands and forces the liquid in the left-hand column down, and in the right-hand column up. This is a very sensitive arrangement, and indicates the presence of very small differences of temperature, for which purpose it is very useful.

Constant Volume Gas Thermometers. — We shall see in Chapter IV. that for standard purposes liquid thermometers are of little value, and gas thermometers alone are used. Although we cannot yet consider these fully, we will now proceed to a general outline of the principle employed in this type of thermometer.

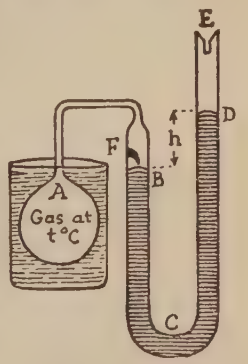

FIG. 6. — Constant Volume Gas Thermometer.

Instead of measuring the expansion of gas caused by a rise in temperature, we *prevent* gas from expanding by applying an opposing pressure. Instead, then, of a relation between the temperature and the expansion it produces, we obtain a relation between the temperature and the pressure applied to prevent the expansion, and to maintain a constant volume. To every pressure there corresponds a definite temperature of gas, and thus we have what is called **a constant volume gas thermometer.** Fig. 6 illustrates the principle of the apparatus—details will be considered in Chapter IV. The gas is contained in a bulb A, connected, as shown, to a U-tube BCD, in which is mercury. Projecting from the tube at F is a horn-shaped index, and the level of the mercury in the left-hand limb is always maintained so as just to reach the extremity of this index. In this way the volume of the gas keeps constant.

The gas is heated in a bath. It tends to expand and to force the mercury down, but to prevent this some more mercury is introduced through the thistle funnel E. If the difference in the two levels h is added to the atmospheric pressure as indicated by a barometer, we get the pressure corresponding to the temperature of the gas. By plotting a curve or drawing up a table of results, the instrument is ready for use as a thermometer.

One of the first to use a constant volume gas thermometer was *Guillaume Amontons* (1663–1705), a Frenchman. He became deaf whilst still a youth, but instead of regarding this as an affliction, he considered it an advantage as ensuring him from interruption in his experiments. Amontons is best remembered as the first scientist to use a thermometer whose upper fixed point was the temperature of boiling water. This was in 1702, although the suggestion for this choice of fixed point was made long before by *Huyghens* (1629–1695) and Newton.

Liquid Thermometers.—It will be noticed that the arrangement shown in Fig. 6 is not a portable one. It is therefore inconvenient for everyday use. For this purpose liquid thermometers are almost entirely in vogue. We come, therefore, to a detailed consideration of liquids as thermometric substances.

Choice of Liquid.—For ordinary purposes the two liquids used, almost exclusively, are mercury and alcohol. Of the two mercury is preferable. It was introduced by *Gabriel Daniel Fahrenheit* (1686–1736), a German scientific instrument-maker. Mercury had been used long before by *Ismael Boulliau* (1605–1694), but Fahrenheit took up the suggestion in 1714, and ensured its general adoption by scientists. The modern thermometer may therefore be considered to date from this time.

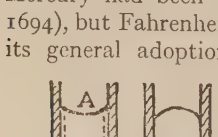

Fig. 7.

The chief advantage of mercury over alcohol is that it does not wet the sides of the tube. Suppose, for example, an alcohol thermometer suffers a fall in temperature (Fig. 7). The reading appears to have fallen from A to B, whereas, since some of the liquid has been clinging to the sides of the tube, the true reading should be C, a little distance above B. This does not happen in the case of mercury, which instantaneously

records the true change in temperature. A further advantage is that mercury is very sensitive, and a small amount of heat produces an appreciable expansion. Add to this the fact that it is easily obtainable in a state of great purity, and it will be seen why mercury thermometers are so popular.

FIG. 8.—Ordinary Modern Mercury Thermometer, with Capillary Tube.

Construction of Mercury Thermometer.—The modern thermometer consists of a bulb, generally cylindrical, connected with a long stem having a fine capillary tube (Fig. 8). It is highly desirable that as far as possible this tube should be of uniform bore, and the first step in the construction of the thermometer is to test the tube to see if this condition is

FIG. 9.—Method of testing the suitability of a Tube for a Thermometer.

satisfied. A small thread of mercury is introduced into the tube, and its length AB is carefully read off by means of a microscope M (Fig. 9). The thread is now pushed a little farther along the tube, and its length read off again. Variations in this length indicate variations in the cross-section of the tube, and if it is not uniform within reasonable limits, it is rejected, and another tested until a suitable tube is obtained. Even then, for very accurate work, a correction curve is drawn up for the thermometer when completed.

A tube having been chosen, a bulb is blown at its lower end, whilst its upper end is formed into a cup-shaped opening with a

constriction C just below it (Fig. 10). Mercury is poured into the cup, but the bore is too fine to permit the liquid to run down to the bulb. It therefore "sits" on the constriction, and has to be forced down by repeatedly heating the bulb slightly above the highest temperature for which the thermometer will be required, thus driving the air out. This has to be repeated several times, the mercury descending in drops each time, until the bulb and stem are full of hot mercury. The top is then sealed off at the constriction C, and on cooling the mercury descends.

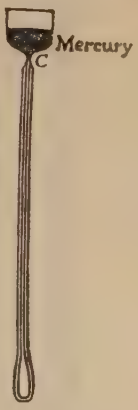

FIG. 10.

The thermometer should now contain nothing but mercury and a little mercury vapour, but as a matter of fact most cheap thermometers are not completely exhausted of air; a small trace is usually left. This, however, is by no means a serious fault.

Finally, the thermometer is strongly heated in an oven for about twenty-four hours at its highest temperature. This reduces serious defects in the glass, to which reference will presently be made.

FIG. 11.—Freezing-point on Thermometer.

The Fixed Points. — Having filled the thermometer, the next step is to mark the fixed points (the melting-point of ice, and the boiling-point of water) upon it.

To determine the melting-point of ice, the thermometer is placed in ice-shavings, or, better still, in clean snow (Fig. 11), and is left there for half an hour or so.

When the reading attained is seen to be quite constant, its position on the stem is noted, and a mark is made.

To determine the boiling-point of water it is necessary to keep the thermometer well immersed in steam arising from boiling water. The experiment must be carried out at atmospheric pressure. This is due to the fact that *differences in pressure affect the boiling-point.* This was first

discovered by Robert Boyle, to whom we have already referred. In 1665 he published a work called *New Experiments and Observations touching Cold*, in which he describes how he introduced into the receiver of an exhaust pump some water whose temperature was somewhat lower than the normal boiling-point. On reducing the pressure in the receiver he was able to reach a point at which the liquid started to boil, so showing that reduction in pressure lowered the boiling-point.

Accordingly in the determination of the upper fixed point, provision must be made for the steam to escape as fast as it is formed—otherwise its pressure will increase, and this will have the effect of raising the boiling-point.

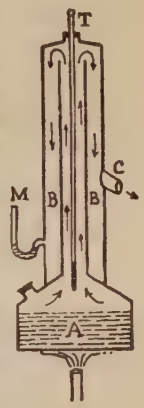

FIG. 12.

The apparatus therefore takes the form shown in Fig. 12. A copper vessel A containing water is surrounded by an air-jacket BB. Leading from the air-jacket is the steam outlet C, whilst a small manometer M indicates the pressure. The thermometer T does not dip into the water at A, but is over it. When the water boils, the bulb and stem are completely surrounded by the steam, which moves upwards in the direction indicated by the arrows, and escapes through the outlet C. The mercury level is seen just above the cork, and when it is observed to be steady, its position on the stem is carefully noted.

Standard Conditions for Boiling-Point of Water.—Since the variations in the atmospheric pressure affect the boiling-point, it becomes necessary to specify the exact conditions under which the boiling-point of water is taken as a standard temperature. If these standard conditions are not present, a correction must be made.

The standard conditions for the fixed points are :

1. A pressure of 760 mm. of mercury.
2. The temperature of the mercury column must be that of melting ice.
3. Observation must be on the sea-level ;
4. And at latitude 45° N. or S.

The need for these conditions arises from the following considerations :

1. For every centimetre increase in the barometric pressure the boiling-point of water is raised 0°·35 C. (see Chapter VII.).

2. Because mercury expands on heating, its density decreases ; this affects the barometric pressure, since pressure $= hg\rho$, where h is the height, g the value of gravity, and ρ the density.

3. g has not a fixed value, but increases as we approach the centre of the earth, and decreases as we get farther from it. Hence we adopt the sea-level as a standard distance from the centre of the earth.

4. The earth is not a sphere, but an oblate spheroid, *i.e.* the polar diameter is shorter (by some 26 miles) than the equatorial diameter. Consequently, even on the sea-level, distances from the centre of the earth vary according as we are near the Poles or near the Equator. Hence it is necessary to specify a definite parallel of latitude, and latitude 45° N. or S. is therefore chosen.

Systems of Graduation.—For scientific purposes, the system of graduation universally adopted is that known as the Centigrade system, and was first used by *Andreas Celsius* (1701–1744) in 1742 at Upsala University, Sweden. The melting-point is called zero, and is marked 0° C., and the boiling-point is called 100° C., though, curiously enough, Celsius originally called the boiling-point 0° and the melting-point 100°. The intermediate space is carefully divided up into 100 divisions. These are divisions of equal volume, and only become equal in length when the bore is perfectly uniform throughout its whole length.

In addition to their scientific utility, Centigrade thermometers are used for ordinary purposes in most countries in which the metric system has been adopted. In this country, however, for household purposes, and also for commercial purposes, the Fahrenheit thermometer is the most frequently used. On this scale the temperature of melting ice is taken as 32°, and the boiling-point is marked 212°, the intervening space being therefore divided up into 180 volume divisions. This is a very cumbersome scale, but nevertheless it is widely used. The zero of this scale is the temperature of a freezing mixture of ammonium chloride and snow, but whether this is why Fahrenheit adopted it or not

is doubtful. The only value that he did deliberately adopt was the temperature indicated by his thermometer when held in the mouth or armpit of a healthy person. This he called 96°, and it just happened on this basis that the boiling-point of water came out at 212°. The normal body temperature on the Fahrenheit scale we now know to be 98°·4, and not 96°.

The Réaumur scale, having the two fixed points for the freezing and boiling of water as 0° R. and 80° R. respectively, is used extensively in Germany and Austria and in some parts of Russia, but in this country it is practically non-existent outside text-books.

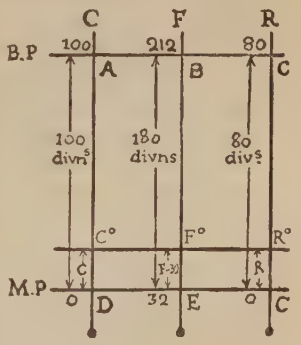

FIG. 13.

Relation between the Scales.—The conversion of temperatures from one scale to another is a simple but important matter. Imagine the three types of thermometers dipping into the same substance, so that the Centigrade thermometer records C.°, the Fahrenheit thermometer F.°, and the Réaumur R.° (Fig. 13). Then these are corresponding temperatures. Consider the interval between the melting- and the boiling-points. We see that 100 Centigrade divisions = 180 Fahrenheit divisions = 80 Réaumur divisions. Obviously the condition for equal temperature is that

$$\frac{AD}{DC^\circ} = \frac{BE}{EF^\circ} = \frac{CG}{GR^\circ}$$

i.e., that $\frac{100}{C.} = \frac{180}{F.-32} = \frac{80}{R.}$

(∵ F.° on the Fahrenheit scale is F.−32 divisions above the melting-point of ice),

i.e., that $\frac{C.}{5} = \frac{F.-32}{9} = \frac{R.}{4}$

E.g., At what temperature will the Fahrenheit and Centigrade readings coincide ?

$$\text{We have } \frac{C.}{5} = \frac{F.-32}{9}$$

$$\text{Cross-multiply, } \therefore 9\,C. = 5\,F. - 160$$

$$\therefore 9\,C. - 5\,F. = -160$$

$$\text{But } F. = C., \ \therefore 9\,C. - 5\,C. = -160$$

$$\therefore 4\,C. = -160$$

$$\therefore C. = F. = -40^\circ$$

Defects in Mercury in Glass Thermometers. — Mercury thermometers are subject to certain defects which in accurate work are a source of annoyance unless provision has been made to reduce them to a minimum. The errors are almost entirely due to the glass, and not to the liquid.

The Secular Rise of the Zero.—When glass is subjected to a fairly high temperature, as happens when a thermometer is being made, on subsequent cooling it does not at once contract to its original volume. Consequently for a long time after manufacture the bulb continues very slowly to contract, thus making what was originally the zero division too low. In consequence, there occurs in the zero of the instrument a gradual rise which may continue for many years. This was first noticed by *Flaugergues* in 1822. The zero of a thermometer made in 1844 by *Dr. Joule* (1818-1889) of Manchester, a brilliant scientist to whom we shall frequently refer, for observations of this *secular rise*, as it is called, is appreciably higher now than it was then. The following record of his figures are interesting: April 1844, 0° F.; February 1846, 0°·42 F.; January 1848, 0°·51 F.; February 1853, 0°·68 F.; April 1856, 0°·73 F.; December 1860, 0°·86 F.; March 1867, 0°·90 F., February 1870, 0°·93 F., February 1873, 0°·94 F.; January 1877, 0°·978 F.; November 1879, 0°·994 F.; December 1882, 1°·020 F.

Glass has no definite physical structure such as is possessed by crystalline bodies, and at most temperatures it shows a certain amount of plasticity, like pitch. It is in order to reduce this error to a minimum that the thermometer, when made, is annealed by being baked and then slowly cooled; but for accurate working the instrument should occasionally be re-standardised.

Correction Curve for Liquid Thermometers.—The ordinary cheap form of mercury or alcohol thermometer in daily use is rarely correct, and for accurate work a correction should be applied to all readings taken. This is best done by means of a correction curve. As a first approximation this can readily be

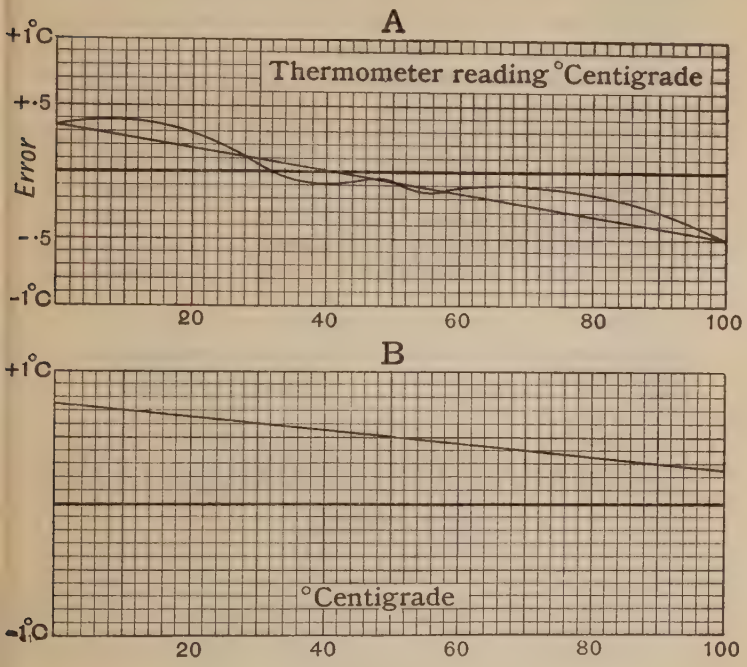

FIG. 14.

obtained by determining the errors at 0° C. and 100 °C. and plotting a straight-line curve, connecting error with temperature reading.

Two examples are shown. In thermometer A (Fig. 14), a positive correction is necessary at the zero reading, and a negative correction at the 100° reading. It will be seen that at one point no correction is necessary. In B, the, correction is positive throughout.

This is only an approximate correction; comparison at a large number of temperatures with a standard thermometer may show a correction curve like the irregular line in Fig. A, but for ordinary purposes the " straight-line " curve is sufficient.

The National Physical Laboratory undertakes the testing of thermometers for a small fee, and supplies the necessary curve.

Special Forms of Mercury Thermometers.—For meteorological and other purposes, it is often necessary to know what have been the maximum and minimum temperature readings recorded by an instrument during any day. For this purpose some self-recording device must be adopted which renders the continual presence of an observer unnecessary. In minimum thermometers the liquid used is alcohol, and the tube is kept horizontal. A small steel dumb-bell-shaped index is introduced as shown in Fig. 15A. The meniscus curving inwards, the surface tension drags the index back with the liquid when the temperature drops,

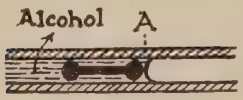

FIG. 15A.

FIG. 15B.

but leaves it behind when it rises. Hence the position A of the index at the end of the twenty-four hours indicates the lowest temperature reached during that interval.

In maximum thermometers mercury is used (Fig. 15B). The meniscus is now directed outwards, so that the index is moved with the column on a rise in temperature, but is left behind when the temperature falls. Hence the maximum temperature for any interval is recorded. The index can always be brought back into contact with the liquid by means of a magnet.

Six's Self-Registering Thermometer.—This form, deservedly popular, gives both maximum and minimum readings. It is illustrated diagrammatically in Fig. 16. The tube is bent as shown, and terminates in two bulbs A and B. A contains alcohol, which reaches round the tube to C. From C to D the tube contains mercury, whilst from D to B, partly filling the upper bulb, we have alcohol again. Above D and C are the two indices. On a rise in temperature, the liquid

in A expands, and pushes the mercury column round. This forces the index *e* up, but the index *f* is left behind. Index *e* therefore records maximum temperatures. When the temperature falls, the liquid in A contracts, causing the column DC to recede. Index *e* is left behind in the alcohol, but index *f* is forced back at each recession. Hence index *f* registers the minimum temperature for any interval.

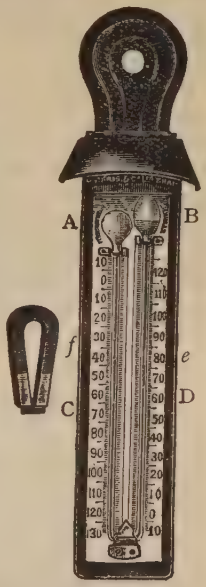

FIG. 16.—Self-Registering Thermometer.

Clinical Thermometers.—For medical purposes it is often required to record the temperature of the human body. On removing the thermometer to the cold air, the thread would at once fall. To prevent this, the thermometer is provided with a small constriction just above the bulb. On heating, the mercury is easily forced past the constriction, but is unable to return without being shaken. Accordingly, the doctor is able to read the temperature recorded without inconvenience. Clinical thermometers register only a small range of temperature—say 95° F. to 110° F.—and are therefore very small and portable.

Extension of Range of Temperature Readings.—Range of Mercury Thermometers.—Mercury solidifies at a temperature of −38°·8 C., and boils at 356°·7 C. An ordinary mercury thermometer is therefore safely able to record temperatures up to 300° C., but is of little use for low temperatures.

Extension of Range Downwards.—When we wish to measure low temperatures, some other liquid having a lower freezing-point than mercury must be used. Alcohol has a freezing-point of −130° C., and therefore is very suitable for low-temperature thermometry. Other liquids which are finding favour are toluene, petroleum, ether, and pentane.

Extension of Range Upwards.—For this purpose we have a special bulb fitted at the upper extremity of the ordinary mercury

thermometer, and over the liquid we introduce an atmosphere of nitrogen under great pressure. This increased pressure has the effect of considerably raising the boiling-point of mercury, and hence of extending the range of its utility as a thermometric liquid.

Fery's Spiral Pyrometer.— For the measurement of high temperatures, liquid-in-glass thermometers are useless, and we have to look for other means. One of these is *Féry's Spiral Pyrometer*. This resembles the thermometers we have already described in that it depends upon the expansion effect of heat upon substances, but in this case the liquid is replaced by a spiral strip made of two metals. The principle upon which this instrument depends is that heat affects different substances differently, so that the same increase in temperature will make two different

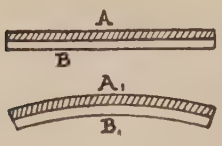

FIG. 17A.

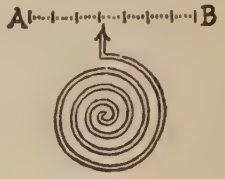

FIG. 17B.

metals of initially identical dimensions expand by different amounts. If we have two equal strips A and B (Fig. 17A) firmly joined together, such that heat makes A expand more than B, then on applying heat to the arrangement, since the two strips cannot be separated, they are forced into the form A_1B_1, *i.e.* they are bent into an arc, that metal which expands more being on the outside of the arc. Similarly, if we have a coil of two different metals (Fig. 17B), then when heat falls on the spiral it either uncoils or coils up, according as the metal with the greater amount of expansion is on the inside or the outside of the strip. In this way a pointer attached to the spiral may be made to move over a graduated scale, and so record the temperature. The arrangement of the instrument is shown in Fig. 18. The spiral A is placed at the focus of the large concave mirror BC, the adjustment being made by means of the adjustment screw E. The pyrometer is attached to the furnace whose temperature is re-

quired at KK. Heat rays fall on the mirror BC, and are reflected to the spiral at its focus A. Any rays which do not quite reach A are caught by a small concave mirror H, and are reflected back to the spiral. The pointer G is of aluminium, and moves under a graduated scale on the window F. D is an eye-piece for focussing

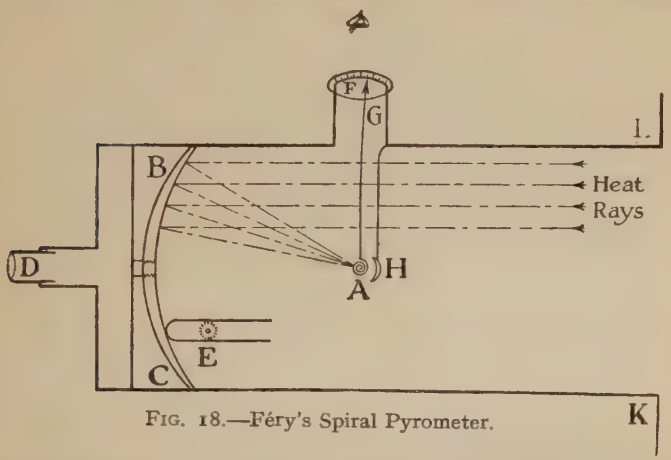

FIG. 18.—Féry's Spiral Pyrometer.

the instrument on to the radiating body, which is rendered visible by means of a small mirror (not shown) near the front of the spiral. The dimensions of the spiral are of the order $\frac{1}{8}$ inch diameter, and $\frac{5}{64}$ inch wide, and the usual range of the instrument is from 500° C. to 1100° C., at which point its error is from 1 to 2 per cent.

EXAMPLES ON CHAPTER I

1. Establish the relations between the standard scales for liquid thermometers.

   Convert (*a*) 13° C., 45° C., 118° C., to Fahrenheit readings.

   (*b*) 15° F., 43° F., 94° F., to Centigrade readings.

2. At what temperature will the Fahrenheit and Centigrade scale readings coincide ?

3. The difference of the Fahrenheit and the Centigrade scale readings of a certain temperature above the freezing-point is 50. Show that their sum is 95.
4. Describe the construction and mode of action of a mercury-in-glass thermometer, and explain how you would test the accuracy of one.
5. In virtue of what properties would you select a liquid for use in a thermometer, and what would determine the dimensions you would give to the instrument ?
6. What is meant by a scale of temperature, and upon what does the definition of any particular scale depend ?
7. Describe how a mercury thermometer should be constructed so that it may be capable of detecting small changes in temperature. How would you graduate such a thermometer with a Fahrenheit scale ?
8. Two thermometers, in one of which is mercury, and in the other air, necessarily agree at the two standard points, but they do not agree at any other temperature. Have you any reason to regard either as giving a truer indication of temperature than the other ?
9. Distinguish carefully between heat and temperature. How may temperatures above 500° C. be measured ?
10. Two mercury thermometers are constructed of the same glass. The internal diameters of their bulbs (which are spherical) are 8 mm. and 6 mm. respectively, whilst their tubes are 3 mm. and 2 mm. internal diameters respectively. Compare the lengths of their degree graduations.
11. Describe some form of self-registering thermometer, and explain its action.
12. What do you understand by the zero error of a liquid-in-glass thermometer ? Explain its cause and remedy.
13. Give an account of the Féry radiation pyrometer.
14. The Appendix shows a list of the melting-points of a number of substances in degrees Centigrade. Compile a corresponding list in degrees Fahrenheit.
15. The Appendix shows a list of the boiling-points of a number of substances in degrees Centigrade. Compile a corresponding list in degrees Fahrenheit.
16. Draw a graph that will enable you to make speedy conversions between Fahrenheit and Centigrade scales. How many points are needed to draw such a graph, and why ? Under what circumstances would you (*a*) be prepared to use the graph, and (*b*) reject the graph in favour of calculation ?

CHAPTER II

THE EXPANSION OF SOLIDS

The Expansion of Solids.—We shall now consider in detail the expansion effect of heat on substances. In this chapter we will confine ourselves to solids, leaving liquids and gases for subsequent study. The actual amount of expansion for ordinary lengths of substance and for ordinary ranges of temperature is very small indeed, and so special apparatus has to be devised to enable exact measurement to be made. Some of these we will now describe.

Weedon's Apparatus.—One of the simplest and yet most accurate forms of apparatus is that due to Weedon. It is illustrated in Fig. 19.

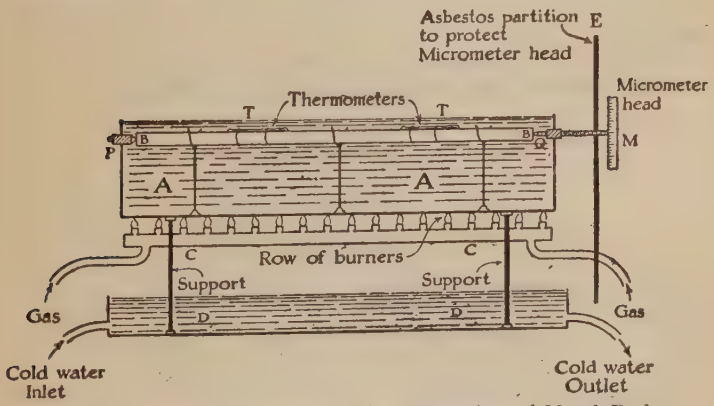

FIG. 19.—Weedon's Apparatus for Expansion of Metal Rods.

The bar BB to be tested is exactly 100 cms. long at 0° C. It is placed in a long zinc trough AA containing oil or water, and is

held on supports between a fixed stop P at one end and a delicate micrometer screw MQ (reading accurately to one ten-thousandth of a centimetre) at the other end. The temperatures are read by thermometers TT attached to the rod. The heat is supplied by a bar-burner CC having a double row of gas-jets along its length, the gas being supplied through inlets at the ends. To prevent errors due to the heat radiated from the trough and the burner, a cold-water bath DD, into which the supports of the upper bath are fixed, maintains a constant circulation of cold water, whilst an asbestos screen E affords the necessary protection to the micrometer screw-head M. The experiment is carried out as follows: The liquid in the trough is first heated to a temperature of about 90° C., the micrometer-head M having been screwed back to allow for the necessary expansion, which must take place entirely to the right, the stop P being fixed. The gas is now withdrawn, and the bar begins to cool. We carefully note the temperature, and screw up M until Q just grips the bar, when we note the micrometer reading. The bar is then allowed to cool to, say, 80° C., when a further reading of the micrometer is taken. Observations are thus made at intervals of, say, 10° C., the resulting contractions being the differences in successive micrometer readings.

This is a very accurate method, and gives good results.

Tube Method.—A simple laboratory method suitable for

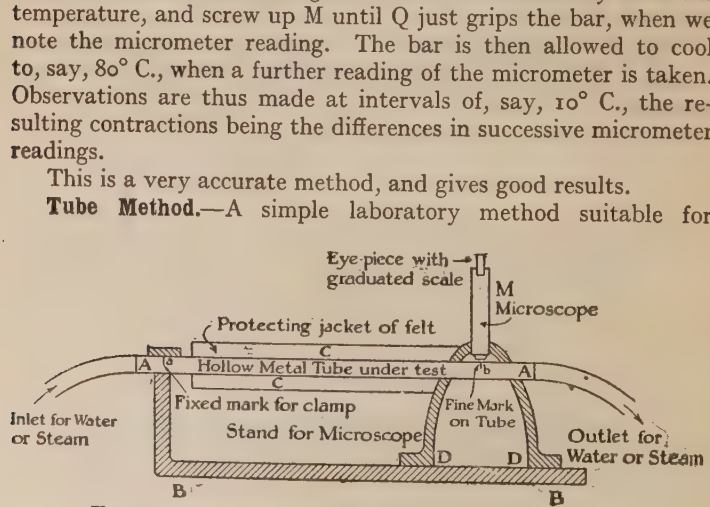

FIG. 20.—Apparatus for Expansion of Hollow Tube.

measuring the expansion of tubes is shown in Fig. 20. The hollow metal tube AA is securely clamped at *a* to a firm metal base BB At the other extremity is a fine mark *b*, and this is observed by

means of a microscope M fitted with a graduated eye-piece, and mounted on a stand DD. A projecting jacket of felt CC surrounds the tube, and prevents losses due to radiation, etc. Ice-cold water of known temperature is first passed through the tube, and the position of the mark *b* is read on the eye-piece scale. Steam is then passed through, and the tube expands to the right, the end *a* being rigidly fixed. A new eye-piece reading is noted, and the difference gives the expansion produced.

Photographic Method.—The following ingenious but exceedingly simple photographic method has recently been devised by the Rev. E. S. Phillips. A long hollow metal tube B (Fig. 21) is attached to a baseboard A which can conveniently be fixed to

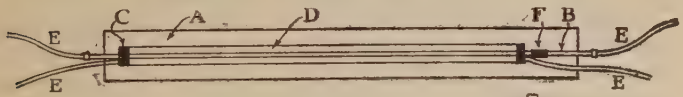

FIG. 21.

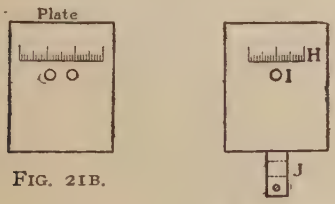

FIG. 21B.

FIG. 21A.

A, baseboard; *B*, brass (or other metal) tube; *C*, stop, holding one end fast; *D*, jacket to run cold water and (afterwards) steam; *E*, rubber tubes to lead in water and steam; *F*, screen, on which is ruled a scale (mm.) and perforated by hole *I*, and clipping on to tube at *J*; *G*, camera with short-focus lens, and length of box about 4 feet.

a wall. A fixed stop C holds one end of the tube firmly, so that all expansion will take place towards the other end. Surrounding the whole length of the tube is a jacket D with inlet and outlet pipes E. Clipped on to the tube, just beyond the extremity of the jacket, is a screen F on which is ruled a millimetre scale H (see enlarged drawing of screen, Fig. 21A). It is perforated by a small hole I. In front of this screen is a long camera-box (about 4 feet) with a short-focus lens.

The procedure of the experiment is as follows :

1. The screen F is first illuminated in front and photographed

2. Cold water at known temperature is now run through both tube and jacket. A light is put behind the screen, and the light through the hole is photographed on the same plate as before.

3. Lastly, steam is sent through the tubes, and another photo of the hole is taken on the same plate. The result on developing the plate is an enlarged view of the scale, and two positions of the hole (Fig. 218). The photograph gives exactly the magnification of the camera and the amount of expansion due to the length from the fixed stop C to the screen. The calculation involved is simple. This method is a very accurate one, its great advantage being the reduction of personal errors to a minimum.

Results for Linear Expansion.—As a result of the various methods described for the measurement of the linear expansion of solid bodies, it is found that for any one substance the increase in length per degree rise in temperature for 1 cm. is always the same. This quantity is spoken of as the **coefficient of linear expansion,** and it differs in value for different substances. On the Centigrade scale, the following may be taken as fairly reliable values for α, as we usually denote this coefficient :

Aluminium	0·000023	Iron	0·0000122
Brass	0·0000188	Lead	0·000029
Copper	0·0000172	Oak	0·00005
Gold	0·0000144	Platinum	0·0000089
Glass	0·0000085	Silver	0·000019
Invar (Nickel Steel)	0·00000087	Steel	0·000012
Zinc	0·000029		

Remembering that a degree Fahrenheit is five-ninths of a degree Centigrade, it will be clear that the coefficient of linear expansion per degree Fahrenheit for any substance is five-ninths of the value per degree Centigrade.

Example 1.—*A brass rod is 84·2 cms. long at 0° C. What is its length at 50° C.?*

Solution.—α for brass = 0·0000188.

∴ 1 cm. of brass will expand 0·0000188 cm. per degree rise in temperature.

$\therefore$ Expansion of 1 cm. for 50° rise in temperature
$= 50 \times 0{\cdot}0000188 = 0{\cdot}00094$ cm.

$\therefore$ Expansion of 84·2 cm. for 50° rise in temperature
$= 0{\cdot}00094 \times 84{\cdot}2 = 0{\cdot}079148$ cm.

$\therefore$ Length of rod at 50° C. $= 84{\cdot}2 + 0{\cdot}079148 =$ **84·2791 cms.**

Example 2.—*A steel ball, 5·001 cms. diameter at zero temperature, rests on a copper ring 5 cms. diameter. At what temperature will the ball just go through?* (α *for steel* $= 0{\cdot}0000011$, *and for copper* $= 0{\cdot}000017$.

Solution.—Let t be the required temperature.

New diam. of steel ball $= l\ (1 + \alpha t)$
$= 5{\cdot}001\ (1 + 0{\cdot}0000011t)$
$= 5{\cdot}001 + 0{\cdot}00005501 1t$ . . . (i)

New diam. of copper ring $= l'_o\ (1 + \beta t)$
$= 5\ (1 + 0{\cdot}000017t)$
$= 5 + 0{\cdot}000085t$ . . . . (ii)

These must be equal.

$\therefore 5 + 0{\cdot}000085t = 5{\cdot}001 + 0{\cdot}000055\ t$ approx.

$\therefore \quad 0{\cdot}00003t = {\cdot}001$

$$\therefore t = \frac{0{\cdot}001}{0{\cdot}00003} = \frac{100}{3} = 33{\cdot}3^\circ \text{ C.}$$

It should be noted that although in the above two examples the original temperature was 0° C., the theory is exactly the same between *any* two temperatures.

Example 3.—*A bar of steel is 45 cms. long at 20° C. Find its length at 90° C., taking* α *for steel* $= 0{\cdot}000012$.

Solution.—Rise in temperature $= 90 - 20 = 70^\circ$ C.

$\therefore$ Expansion of 1 cm. for 70° rise in temperature
$= 70 \times 0{\cdot}000012 = 0{\cdot}00084$ cm.

$\therefore$ Expansion of 45 cm. for 70° rise in temperature
$= 0{\cdot}00084 \times 45 = 0{\cdot}03780$ cm.

$\therefore$ Length of steel bar at 90° C. $=$ **45·038 cms.**

Formula for Expansion.—It is possible to express the relationship between the lengths of a substance at any two temperatures by a simple formula.

If l_o is the length of a bar at the **lower** temperature,
and l_t is the length of a bar at the **higher** temperature (t° above the lower),
and α is the coefficient of linear expansion of the material,
then the expansion of one unit length is αt for the total rise in temperature.

Hence unit length at the lower temperature becomes $(1+\alpha t)$ in length at the higher temperature.

$\therefore$ l_o units of length at the lower temperature become $l_o(1+\alpha t)$ units of length at the higher temperature, i.e., $l_t = l_o(1+\alpha t)$.

As a matter of fact, this formula does not completely represent the facts of linear expansion. The deviations from the exact values are, however, so exceedingly minute for ordinary ranges of expansion that they need not be considered here.

The student is not recommended to use this formula in working out questions and examples. It is far better to work from common sense and first principles, as in the case of the worked examples given above. We merely quote the formula here in order to deduce from it certain other important facts regarding the expansion of areas and volumes. To these we now proceed.

Superficial Expansion.—Consider a square plate whose side is unit length at 0° C. (Fig. 22), and its area 1 square unit. Suppose that the plate is heated to a temperature of 1° C., and that the coefficient of linear expansion of the material of the plate is α. The plate will expand and will become a larger square whose side is $(1+\alpha)$ units. The new area will be $(1+\alpha) = 1+2\alpha+\alpha^2$, and the amount of *area expansion* is therefore $2\alpha+\alpha^2$. This is made up, in the figure, of three portions—two rectangular strips, each α square units, and the tiny corner square whose area is α^2. Actually this tiny corner may be ignored, because, as compared with α, it is minute and negligible. For example, if $\alpha = 0{\cdot}000012$, α^2 becomes $0{\cdot}000000000144$, and if we are working to six decimal places this is in effect zero.

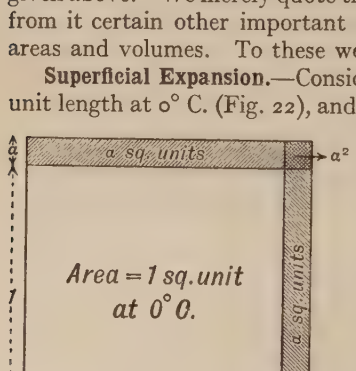

FIG. 22.

Hence, if the linear expansion of unit length is α, the area or superficial expansion of unit area is 2α, and if we call this β, we may express the result by the statement that **the coefficient**

of superficial expansion is twice the coefficient of linear expansion for the same material.

Corresponding to $l_t = l_o\,(1+\alpha t)$ for lineal expansion,
we have $\mathbf{A}_t = \mathbf{A}_o(1+\beta t)$ for superficial expansion,

where A_t = area at $t°$ higher than the temperature at which the area is A_o.

Cubical Expansion.—In exactly the same way we may extend the previous paragraph to the case of *volume or cubical expansions*, and by considering a unit cube each of whose edges is expanded by α on being heated 1°, we can show that the total volume expansion correct to the first six or seven places of decimals is 3α, and if we call this γ, then γ, **the coefficient of cubical expansion of a substance, is three times the coefficient of linear expansion for the same material.**

Using similar nomenclature to the above, we have

$$\mathbf{V}_t = \mathbf{V}_o(1+\gamma t),$$

and the relation between the three coefficients is therefore expressed by

$$\alpha : \beta : \gamma = 1 : 2 : 3.$$

Temperature and Density.—When we heat a body, its volume increases. Its weight, however, remains unaltered, and so the density diminishes. It is important to know the exact connection between the densities ρ_t and ρ_o of a body at two temperatures, one of which is $t°$ higher than the other. The relationship depends upon the fact above referred to, that when the volume increases the density decreases; *i.e.*

$$\frac{\rho_t}{\rho_o} = \frac{V_o}{V_t}$$

But $$V_t = V_o(1+\gamma t),$$

$$\frac{\rho_t}{\rho_o} = \frac{V_o}{V_o(1+\gamma t)} = \frac{1}{(1+\gamma t)}$$

If we divide 1 by $(1+\gamma t)$ by the ordinary methods of algebraic division, we get what looks like a laborious and never-ending result, $1-\gamma t+\gamma^2 t^2-$, etc. But the first two terms are all that

matter. Every other term is of a higher power of γ, and for the reasons already given are negligible. So we can write $(1-\gamma t)$ in the place of $\frac{1}{1+\gamma t}$ in the above, with the enormous advantage of avoiding a cumbersome division sum every time we want to calculate a density at one temperature from that at a lower temperature.

I.e. $$\frac{\rho_t}{\rho_o} = 1-\gamma t,$$

or $$\rho_t = \rho_o(1-\gamma t),$$

a result which corresponds to that of the volumes formula, except that we have a minus sign in the bracket instead of a plus. This we should expect, because the density is less at the higher temperature.

Example 4.—*A cylinder of iron 20 cms. long floats vertically in mercury, both being at the temperature of 0° C. If the common temperature rises to 100° C., find by how much the cylinder will sink, given that the specific gravities of iron and of mercury are 7·6 and 13·6 respectively at 0° C., the coefficient of cubical expansion of mercury is 0·000182, and the coefficient of linear expansion of iron is 0·0000118.*

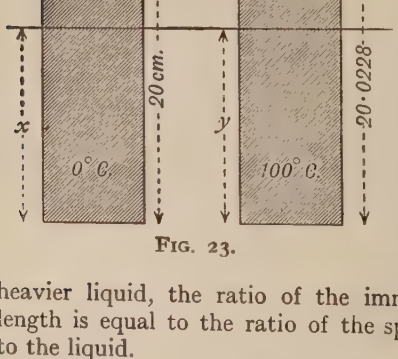

FIG. 23.

Solution.—From hydrostatics, we know that when a solid rod floats in a heavier liquid, the ratio of the immersed length to the total length is equal to the ratio of the specific gravities of the solid to the liquid.

Let x = length immersed at 0° C. (Fig. 23).

Total length = 20 cms. at 0° C.

$$\therefore \quad \frac{x}{20} = \frac{7\cdot6}{13\cdot6}$$

Whence $x = 11\cdot18$ cms.

We have now to carry out a similar calculation at 100° C., but to do so we must know three values :

(i) The length of the rod at 100° C. (l_{100}),
(ii) The specific gravity of the mercury at 100° C., and
(iii) The specific gravity of the rod at 100° C.

(i) Since $l_{100} = l_0(1 + \alpha t)$,
we have $l_{100} = 20(1 + 0{\cdot}0000118 \times 100)$
$= 20 \times 1{\cdot}00118$
$= 20{\cdot}0236$ cms.

(ii) In the formula $\rho_t = \rho_0(1 - \alpha t)$, α is the coefficient of **cubical** expansion, and for iron this equals $3 \times 0{\cdot}0000118 = 0{\cdot}0000354$.

Hence we have the specifie gravity of iron at 100° C. (ρ_{100}).

$= 7{\cdot}6(1 - 0{\cdot}0000354 \times 100)$
$= 7{\cdot}6(1 - 0{\cdot}00354) = 7{\cdot}6 \times 0{\cdot}99646$
$= 7{\cdot}5731$.

(iii) Similarly the specific gravity of mercury at 100° C.

$= 13{\cdot}6(1 - 0{\cdot}000182 \times 100)$
$= 13{\cdot}6 \times (1 - 0{\cdot}0182) = 13{\cdot}6 \times 0{\cdot}9818$
$= 13{\cdot}3525$.

(iv) We are now able to complete the question.
Let y = length immersed in the mercury at 100° C.
Total length at 100° C. = 20·0228 cms.

$$\therefore \frac{y}{20{\cdot}0236} = \frac{7{\cdot}5731}{13{\cdot}3525}$$

$$\text{whence } y = \frac{7{\cdot}5731 \times 20{\cdot}0236}{13{\cdot}3525}$$

$$= 11{\cdot}36 \text{ cms.}$$

$\therefore$ distance through which the rod sinks $= 11{\cdot}36 - 11{\cdot}18$
$= \mathbf{0{\cdot}18}$ **cm.**

The Expansion of Hollow Bodies.—Consider any hollow body (Fig. 24). Let it be cut up into a number of parallel slices, each of whose thickness at 0° C. is l_o. On raising the temperature to $t°$ C., the thickness, which is independent of the hollow itself, becomes l_t, such that

$$l_t = l_o(1 + \alpha t).$$

FIG. 24.—Diagram for Expansion of Hollow Body.

This will be true for all slices, the expansion being perpendicular to the plane of the section, and will remain true whatever the direction of the section. It is therefore true for all directions. Hence we see that the expansion of a hollow body follows the ordinary laws of expansion for solid bodies.

To consider the increase in volume of the cavity, imagine it to be filled with a substance of the surrounding solid. Then the increase will be the same as the increase of the solid, *i.e.*,

$$volume_t = volume_o(1 + \gamma t).$$

The Force of Thermal Expansion.—The forces involved in thermal expansion are very great indeed, and if adequate provision

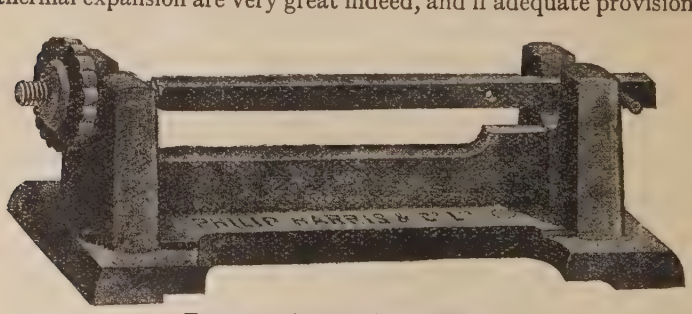

FIG. 25.—Contraction of Metals.

is not made for the expansion consequent upon a rise in temperature, as for example in the familiar case of the spaces left between lengths of railway line, disastrous results will follow. Thus for an average size of steel rail, the force with which two adjoining sections would press against each other for a rise in temperature of

say 30° C., assuming there were no space between them initially, is as much as 5 tons per square inch, *i.e.* about 50 tons force for a rail of 10 square inches in cross-section.

Fig. 25 shows a well-known experiment. A rod fitted with a screw at one end is fitted into a bracket between two uprights. The arrangement is locked by a circular nut at the screw-end and an iron pin passed through a hole in the other end. The whole is vigorously heated, and the rod expands so that the nut becomes loose. Screw this up tightly and allow the whole to cool. The rod contracts, but having no room otherwise, the force of contraction exerted at the cast-iron pin causes it to snap suddenly in two. Example 14 at the end of the chapter illustrates how this idea was once actually applied in Paris to straighten two walls of a building (the Conservatoire des Arts et Métiers). The roof was too heavy, and had caused the walls to bulge and to be in danger of collapsing. Heavy iron stays were passed right through and were tightened up *hot*, and the force of this contraction on cooling straightened the walls.

The Practical Applications of Thermal Expansion.—The previous paragraph shows clearly that we cannot afford to neglect the influence of expansion upon structures. Accordingly provision is made, by the use of various devices, for the free expansion of such structures either at their extremities or at the junction of their parts. Thus a telescopic device is sometimes used at the junction of two lengths of piping, the inner piece being free to expand by rolling on a pin. In Fig. 26 we see the same idea applied to one end of a bridge. The tremendous forces involved in the contraction of hot bodies have been made use of in the manufacture of big guns, one part being shrunk on to another. The same method is employed in the shrinking of metal tyres on to cartwheels. Boiler-plates are firmly riveted together by the contraction of red-hot rivets, which are hammered up tight.

Fig. 26.—Provision for Expansion of Bridgework.

Pendulum clocks are seriously affected by temperature unless

the necessary precautions are taken. The correct working of the pendulum depends upon the constancy of the distance of its centre of gravity from the point of suspension. On a rise in temperature, this point would of course move down. To counteract this, pendulums are "compensated" by being made of two metals. The principle of the compensated pendulum is as follows :

FIG. 27.—To show Principle of the Compensated Pendulum.

Let AB and CD (Fig. 27) be two bars of lengths l and l_1, and of different expansion coefficients α and β. For a rise in temperature of $t°$ C., if A is a fixed point,

AB expands downwards a distance $= l\alpha t$,
and CD expands upwards a distance $= l_1\beta t$.

For these to neutralise each other, we must have

$$l\alpha t = l_1\beta t.$$

i.e.

$$\frac{l}{l_1} = \frac{\beta}{\alpha}$$

i.e., *their lengths must be inversely as their coefficients of expansion.*

In Harrison's gridiron pendulum (Fig. 28) this condition is satisfied by having in all five rods, the middle and the outer two being of iron, and the remaining two of zinc. In another form of pendulum due to Graham, the expansion downwards of a steel rod is counter-balanced by the expansion upwards of two cylinders of mercury, one on either side of the central rod. In the compensated balance-wheel of a good watch, the radius is automatically adjusted by making the rim of two metals, usually brass on the outside and steel inside, the principle being that described in Féry's pyrometer (*see* p. 20). This, with the use of weights, automatically adjusts both the size and elasticity of the wheel, and so preserves the rate of the watch. In Fig. 29 is shown a simple fire-alarm device. In buildings where it is dangerous for a certain temperature to be exceeded, a thermometer is arranged with two platinum wires fused through—one at the bulb and the other at

FIG. 28.—Harrison's Gridiron Pendulum.

the limiting temperature-reading. Mercury being a conductor of electricity, when the "danger" temperature is reached, the circuit is completed, and a bell rings or a lamp flashes out. Another fire-alarm device is shown in Fig. 30. A wire AB

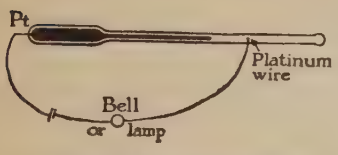

FIG. 29.—Thermometric Fire Alarm.

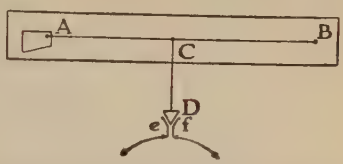

FIG. 30.—Arrangement for Fire Alarm.

has a metallic plug D suspended from its mid-point C. On expansion caused by a sufficiently high (and dangerous) temperature, the wire sags, and D descends and makes electrical contact at *ef*, causing a bell to ring.

EXAMPLES ON CHAPTER II

1. Define coefficient of linear expansion.
   A copper wire is 100 cms. long at 0° C., and 100·17 cms. long at 100° C. What is its temperature when its length is 100·04 cms. ?
2. Describe a method by which the coefficient of linear expansion of a bar or tube of metal may be determined, and point out the chief sources of error, and the difficulties in the method you describe.
3. The diameter of an iron wheel is 5 feet. Find the increase in the circumference when the temperature is raised 300° C. (The coefficient of linear expansion for iron = 0·0000122.)
4. An iron yard-measure is correct at 0° C. Express in inches its error at 100° C. (a for iron = 0·000011.)
5. A bar of iron is 45 cms. long at 20° C. Find its length at 90° C.
6. An iron tube is 1512·45 feet long at 10° C., and 1513·0 feet long at 40° C. Find the coefficient of linear expansion of iron.
7. A pane of glass measures 24·5 cms. by 12·8 cms. at 10° C. What is its area (*a*) at 0° C., (*b*) at 90° C. ? (a for glass = 0·0000086.)

8. Explain how it is possible to use photography for the measurement of linear expansions.
9. Why can a thin platinum wire be sealed into a glass tube so as to make an air-tight joint ?
10. The volume of an iron ship is 10,000 cubic yards at 0° C. What is its volume at 25° C. if the coefficient of linear expansion of iron = 0·000011 ?
11. State some of the industrial applications of the expansion effect of heat on substances.
12. A solid of linear expansibility 0·000015 contains a 500 c.c. cavity in which is a 200 c.c. body of linear expansibility 0·00003. How much does the volume of air space left change per degree ?
13. Explain why it is a sufficiently near approximation to take the coefficient of volume expansion as three times the coefficient of linear expansion. Calculate the percentage error involved in this approximation in the case of the expansion through 100° C. of a body whose coefficient of linear expansion is 0·00001234.
14. Two walls originally perpendicular, and 20 feet apart, have bulged out 1·2 inches. To make them perpendicular again by contraction of iron bars, find the necessary rise in temperature, if the air is at 0° C. (Coefficient of linear expansion of iron = 0·000011.)
15. A metre is the distance between two parallel lines on a platinum bar at 0° C. The yard is the distance between two parallel lines on a brass bar at 18° C. These are compared at 18° C. One metre is found to be equal to 1·09393 yards. If the coefficient of linear expansion for platinum is 0·0000086, and for brass 0·000019, express the metre in terms of the yard.
16. A steel ball 5·01 cms. diameter at zero temperature rests on a copper ring 5 cms. diameter. At what temperature will the ball just go through ? (a for iron = 0·000011, and for copper a = 0·000017.)
17. What is meant by the coefficient of linear expansion of a substance ? How is it related to the coefficient of superficial and cubical expansion ?

    The longerons of an aeroplane are 15 feet in length. If the temperature during flight ranges from 20° C. to 24° C., find the alteration in length of the longerons. (The coefficient of expansion of the wood is 0·000005 per degree Centigrade.)
18. Why do you think the expansion of a metal is disadvantageous in engines ?

    A cast-iron cylinder 10 inches long and $3\frac{1}{2}$ inches diameter has a working temperature of 250° C. Given that the linear coefficient of expansion of iron is 0·0000118,

and that the above dimensions were ascertained at a temperature of 15° C., find (*a*) the increase in the diameter, and (*b*) the increase in the volume.

19. Describe with a sketch (1) a gridiron pendulum, (2) a compensated balance-wheel.

The diameter of a steel shaft is 3 inches at 0° C. What is it at 60° C.? (The coefficient of expansion of steel is 0·000012.)

20. How would you determine the coefficient of linear expansion of a metal rod?

The space between the 100-foot lengths of a steel rail (coefficient of linear expansion = 0·000012) is $\frac{1}{4}$ inch. What is the safe rise in temperature?

21. The density of a piece of zinc is 7·122 grams per c.c. at 42° C., and 7·124 grams per c.c. at 12° C. Find the coefficient of linear expansion of zinc.

22. The length of the flying-wire of an aeroplane is 4 feet 6 inches as measured in England. The machine is then transported to Mesopotamia. The rise in temperature being 29° C., find the increase in the length of the wire if its coefficient of expansion is 0·000019.

23. Give two examples in practice of the need for taking precautions against expansion due to heating, and two where the property of expansion is utilised.

In engine valves there is danger of the valves "seizing up" if insufficient clearance is provided. Assuming, in one type of valve, the clearance between the guide and valve-stem to be 0·002 inch, and that no expansion takes place in the guides, what rise in temperature will cause the valves to seize up? (Diameter of valve stem = $\frac{3}{8}$ inch, and coefficient of expansion = 0·000012 per degree Centigrade.)

24. The testing of circular holes for size is sometimes done by what are called limit gauges. A limit gauge for a standard 3-inch hole is made of steel and is designed to work at 60° F. What will be the error if the gauge is used in a tropical country at a temperature of 120° F.? (The coefficient of linear expansion of steel = 0·000012 per degree C.)

25. A steel collar is to be shrunk on to a shaft, and for this purpose will be heated to 500° F. The shaft itself, which is 3 inches diameter at the workshop temperature of 60° F., may be considered to remain at that temperature during the operation. To what internal diameter should the collar be finished in the workshop so that when heated it may exceed the diameter of the shaft by 0·002 inch? (Coefficient of linear expansion of steel is 0·0000066 per degree F.)

26. When casting, an allowance must be made for shrinkage on cooling. This is usually one-eighth of an inch per foot. If the temperature of the casting is 2250° F., and the final temperature on cooling is 60° F., what coefficient of expansion (or contraction) per degree Fahrenheit does this allowance assume ?

CHAPTER III

THE EXPANSION OF LIQUIDS

The Containing Vessel.—The study of the expansibility of liquids with the application of heat, to which we now proceed, is complicated by the presence of the containing vessel. No liquid can be experimented upon by itself—it must be placed in a solid vessel of some kind, and on applying, say, a Bunsen flame, both vessel and liquid expand. Consequently what we observe is not the true expansion of the liquid, but the true expansion less the expansion of the containing vessel. For suppose we consider the two expansions separately, as of course we may do without prejudicing the final result, *i.e.* suppose, first, that the containing vessel expands alone to its final volume at the higher temperature, and that then the liquid expands to its final volume. The result of the first expansion will be to diminish the level of the liquid by an amount equal to this expansion of the containing vessel, *i.e.* in Fig. 31, from the original level *ab* to the level a_1b_1. Hence the volume aba_1b_1 represents the amount by which the vessel expands. The expansion of the liquid itself, however (equal to the volume represented by $a_1b_1a_2b_2$), now causes its level to rise to a_2b_2. Apparently, therefore, to the observer the expansion has been from the original level *ab* to the level a_2b_2, *i.e.* the volume aba_2b_2; whilst actually its expansion is equal to the volume $a_1b_1a_2b_2$. Hence we see that **the actual or absolute expansion of the liquid is equal to the sum of its apparent expansion and the expansion of the containing vessel.** Dealing

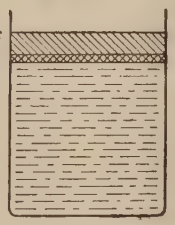

FIG. 31.

with unit volumes of liquid and containing vessel, we therefore have the relation

$$\alpha = \beta + \gamma$$

where α = absolute coefficient of cubical expansion of the liquid, β = apparent coefficient of cubical expansion of the liquid, and γ = absolute coefficient of cubical expansion of the containing vessel.

The Two Methods.—The methods for the determination of the coefficient of cubical expansion of a liquid fall naturally into the following two groups: (1) Relative methods; (2) absolute methods.

In the first group the determination is dependent upon a knowledge of the expansion of the containing vessel, whilst in the second group means are devised to render the observations independent of the containing vessel.

We will describe one example from each group of methods.

Relative Methods—The Weight Thermometer.—This apparatus is illustrated in Fig. 32. It consists of a cylindrical glass bulb about 6 cms. long, coming from which is a capillary tube bent as shown. It is first weighed empty, and then filled with the liquid to be tested by repeated heating and cooling, as in the filling of a thermometer. The final cooling is done with the bulb immersed in an ice-bath, so that the instrument is filled with liquid at 0° C. It is again weighed, and by difference from the initial weighing the mass m of the liquid at 0° C. is obtained. If the density of the liquid at 0° C. be ρ_o, then the volume of the thermometer at 0° C. will be $\frac{m}{\rho_o}$. Now immerse the bulb in an oil-bath at t° C. Both instrument and liquid expand, and some of the latter is forced out. Catch the overflow in a weighed beaker and weigh it. By difference we get the mass m_1 of the liquid which has overflowed. This represents the apparent total expansion of the liquid from 0° C. to t° C. It is not the real expansion, because, owing to the

Fig. 32.—Weight Thermometer.

expansion of the weight thermometer itself, some of the expanding liquid, which would otherwise be forced out, is able to remain in the bulb.

Now at 0° C. the volume of the overflow $= \frac{m_1}{\rho_0}$.

$\therefore$ the apparent expansion of $\frac{m}{\rho_0}$ c.c. of liquid on being heated from 0° C. to t° C. $= \frac{m_1}{\rho_0}$.

$\therefore$ the apparent expansion of $\frac{m}{\rho_0}$ c.c. per degree rise in temperature $= \frac{m_1}{\rho_0 t}$.

$\therefore$ the apparent expansion of 1 c.c. per degree rise in temperature $= \frac{\frac{m_1}{\rho_0 t}}{\frac{m}{\rho_0}} = \frac{m_1}{mt}$.

This, by definition, is the coefficient of apparent expansion of the liquid.

Let β denote the coefficient of expansion of the material of the weight thermometer.

Then we have $\alpha = \frac{m_1}{mt} + \beta$, where α is the required coefficient of absolute expansion of the liquid.

This experiment, in a slightly modified form, can also be carried out by substituting a specific-gravity bottle for the weight thermometer. The necessary modifications should offer no difficulty to the student.

Absolute Methods—Dulong and Petit's Experiment.—We now pass to the group of experiments in which the expansion of the containing vessel need not be considered. The same principle is used throughout.

Two well-known facts of hydrostatics are employed. The first is that if we have a column of fluid at rest of height h and

density ρ, the pressure at its base, *i.e.* the force per unit area of base, is equal to $hg\rho$, where g is the gravity constant.

The second fact is that at any two points in the same horizontal level of a fluid at rest, the pressures are equal. The simplest of the absolute methods was due to Dulong and Petit.

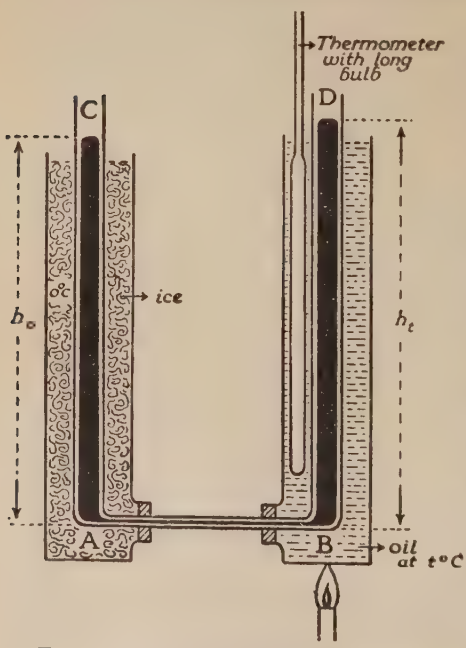

FIG. 33.—Dulong and Petit's Apparatus for Expansion of Mercury.

Alexis T. Petit (1791–1820) was a young Professor of Physics at the Polytechnic School in Paris whose early death robbed science of a worthy servant. His collaborator, *Pierre Louis Dulong* (1785–1838) was also Petit's successor in the Professorship of Physics. Dulong was by profession a physician, but he gave himself up to physical research heart and soul —and indeed pocket, since he practically impoverished himself through the purchase of apparatus. The experiment with which we are here concerned is illustrated in Fig. 33. The essential parts are a U-tube in which the two wide vertical limbs CA and DB are joined by a narrow horizontal tube AB. Each is surrounded by a jacket, that round AC containing ice-shavings at 0° C., and that round BD containing oil or water at $t°$ C. (steam is often used). A long-bulb thermometer is immersed in the latter jacket. The liquid experimented upon, usually mercury, is placed in the U-tube, and, since the right-hand side is at a higher temperature, its

density diminishes, and it will expand to a height greater than that of the mercury column in the left-hand side.

Let h_o = the height of the left-hand column above the base A.

Let h_t = the height of the right-hand column above the base B.

Let ρ_o = the density of mercury at 0° C.,

and ρ_t = the density of mercury at t° C.

Also, let α = the coefficient of cubical expansion of mercury.

Now if A and B are at the same horizontal level, we have

$$h_o g \rho_o = h_t g \rho_t.$$

But $$\rho_t = \rho_o(1 - \alpha t).$$

Substituting, we get $$h_o \rho_o = h_t \rho_o (1 - \alpha t).$$

Whence $$\alpha = \frac{h_t - h_o}{h_t t}$$

It will be seen that the expansion of the containing vessel does not enter into the argument.

The observations are very simple, consisting in a measurement of the two heights and the temperature of the hot limb.

The experiment, however, does not come up to modern requirements. The hot limb is not necessarily of a uniform temperature, the upper ends of the mercury columns are exposed to atmospheric temperature changes, and the columns are widely separated, making a comparison of the heights difficult. Dulong and Petit measured the heights by means of a telescope, but a small tilt in the telescope could lead to serious errors.

Expansion Coefficients for Liquids.—Liquids expand much more than solids. A few typical values are given below.

Alcohol	0·00105	Olive Oil	0·00074
Benzine	0·00124	Petroleum	0·0010
Glycerine	0·00052	Turpentine	0·00105
Mercury	0·000182	Water above 10° C.	0·00043

The absolute method has only been applied to mercury, and more recently, by the German Physical Laboratory, to water. Using the values obtained for these liquids, the dilatometer method is applied to other liquids.

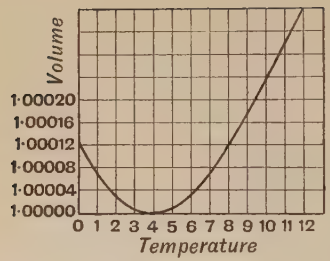

FIG. 34.—Curve showing Behaviour of Water between 0° C. and 4° C.

Behaviour of Water between 0° and 4° C.—Water is a remarkable exception to the general rule of expansion on heating. When water at 0° C. is heated, we find that, instead of an expansion, a contraction takes place. This continues until a temperature of 4° C. is reached, after which further heating produces expansion. The behaviour is clearly shown by the graph (Fig. 34).

The following are exact values obtained in 1900 at the German Physical Laboratory for the volume of unit weight of water :

Temperature.	Volume of Unit Weight.
0° C.	1·0001324
4° C.	1·00000
8° C.	1·0001241

whilst the following table shows the behaviour of water heated above 80° C. :

Temperature.	Volume of Unit Weight.	Density.
0°	1·0001	0·9999
4°	1·0000	1·000
10°	1·0002	0·9998
20°	1·002	0·998
30°	1·004	0·996
40°	1·008	0·992
50°	1·012	0·988
60°	1·017	0·983
70°	1·023	0·978
80°	1·029	0·972
90°	1·036	0·966
100°	1·043	0·960

A possible explanation which has been suggested to account for this remarkable behaviour of water is that at 0° C. it consists chemically not of the normal H_2O molecules, but of "double molecules," *i.e.* of $2H_2O$, occupying rather more volume than the H_2O molecules, and it is the splitting up of these complicated molecules into the simpler ones which accounts for the changes occurring. This splitting up is thought to be a continuation of the process of the melting of ice, *i.e.* the melting process is not in its fullest sense complete until the temperature of 4° C. (or 39°·1 F.) is reached. It should be noted that, in addition to water, there are one or two other substances which under certain conditions contract on heating, as, for example, silica below −80° C., and iodide of silver below 142° C., and iron, to which we shall refer later.

Convection Currents in Liquids.—The experimental investigation of the exact temperature of maximum density of water is dependent upon the phenomenon of *convection currents*, to be fully dealt with in Chapter XII. For the immediate purposes of this chapter, however, we proceed to give the following short explanation of the phenomenon. When heat is applied to a liquid contained in a flask (Fig. 35), initially only the lower layers are warmed, and differences of temperature are therefore set up. This, however, affects the density in the different portions of the liquid, since increase of temperature must produce a decrease in density. The cooler upper layers of liquid therefore become heavier than the warmer lower layers. The latter are thus forced upwards, and are replaced by the heavier and cooler layers. These in their turn become heated to a temperature greater than that of the liquid above them, and, becoming lighter, are also forced to the top.

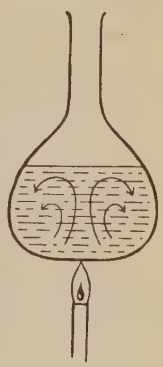

FIG. 35.

In this way a circulation of the liquid is produced, and lasts as long as differences in temperature exist. To this circulation is given the name *convection*.

The Temperature of Maximum Density for Water—(1) Joule's Experiment.—To determine the temperature at which

water attains its maximum density, Joule used the apparatus illustrated in Fig. 36. Two upright cylindrical vessels were connected at the bottom by a horizontal tube fitted with a stop-cock, and at the top by a horizontal channel in which was a floating index.

The stop-cock being initially closed, water was poured into one vessel at a temperature slightly below 4° C., and into the other at a temperature slightly above 4° C. On opening the stop-cock, if the densities are different, the pressures at A and B would be unequal, and a convection current would be set up, which would at once be detected by the movement of the floating index in the channel. But as a matter of fact there are temperatures on either side of the temperature of maximum density such that the densities corresponding to these temperatures are equal, and in this case, on opening the stop-cock, no convection current would be produced, and therefore the index would remain motionless. Accordingly, Joule carefully adjusted the temperatures, and found that when one vessel was at 3°·9 C. and the other at 4°·1 C. the densities were equal. It was therefore reasonable to suppose that the required temperature of maximum density was the mean of these two.

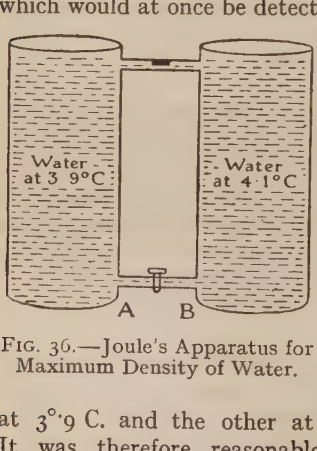

FIG. 36.—Joule's Apparatus for Maximum Density of Water.

He then repeated the experiment with temperatures of, say, 3°·95 C. and 4°·05 C., and so on, contracting the range of temperatures between which the maximum point must lie, until he was within $\frac{1}{50}$° C. of the required result.

This was a most accurate method, the index used being extremely sensitive to the small convection currents set up.

Hope's Maximum Density Apparatus.—This is illustrated in Fig. 37. A vessel D filled with cold water at, say, 8° C. has a "collar" CC round its middle portion, containing a freezing mixture of some kind. Thermometers A and B give the temperatures of the upper and the lower layers of water respectively.

Initially, both show a temperature of, say, 8° C. The temperature in the middle, however, falls owing to a steady cooling produced by the jacket, and the behaviour of the water is normal, *i.e.* it contracts and increases in density. It therefore sinks, setting up convection currents. The thermometer B records a fall in temperature, and the water from the bottom, initially at 8° C., now rises to the collar, to be in its turn cooled and sent down again, still further reducing the temperature reading of ther-

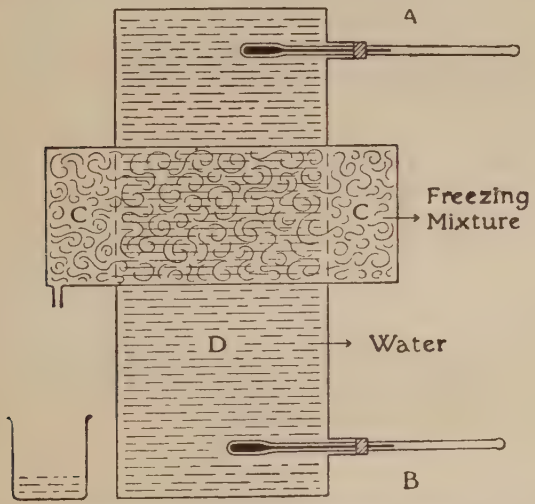

FIG. 37.—Hope's Apparatus for Maximum Density of Water.

mometer B. Thermometer A during this process is very little affected, there being no means by which the cooling is transmitted upwards; but when the lower thermometer reaches 4° C., a change is observed. Further cooling, instead of further increasing the density, now causes the liquid to expand, and therefore decreases the density. The water accordingly rises to the top instead of sinking to the bottom, and it is now the upper thermometer which begins to fall, thermometer B remaining stationary at 4° C. Thermometer A rapidly falls, and the temperature soon goes below 4° C. and reaches zero.

Consequences of the Anomalous Behaviour of Water.—It will now be understood by the student why water, unlike other liquids, freezes from the top downwards. When the atmosphere cools below 4° C. the convection currents are upwards, and so the cooling is unable to be transmitted by convection to the general body of the water, *i.e.* the bottom maintains a uniform temperature of 4° C. Presumably freshwater fish are grateful for this accommodating behaviour of water, since the presence of ice at 0° C. does not prevent them from enjoying a temperature of 4° C. Householders, on the other hand, are not grateful when a thaw shows them the gaping "wound" in the pipes produced by the forcible expansion of the cold water and ice at the previous frost.

Undercooled Water.—In certain circumstances water may be cooled below 0° C. without freezing, and it is then spoken of as *undercooled* or *supercooled water*. The necessary conditions are absolute absence of movement and dust, and a perfectly smooth surface of contact with the containing vessel. When such water is stirred or in any way disturbed, freezing at once sets in, and the temperature rises to 0° C. The expansion which occurs below 4° C. continues for supercooled water. *Wiedner*, in 1866, using the dilatometer method, obtained the following results for the volume of unit weight of supercooled water:

$$4^\circ\ C. = 1{\cdot}000.$$
$$0^\circ\ C. = 1{\cdot}000125.$$
$$-4^\circ\ C. = 1{\cdot}000556.$$
$$-8^\circ\ C. = 1{\cdot}001328.$$

Undercooling is not confined to water, but may occur with many other liquids.

The Effect of Pressure on the Expansion of Liquids.—*Amagat* has investigated experimentally the influence of pressure upon expansion, and, as one would reasonably expect, has found that in general increase in pressure diminishes the expansion. Water is again an exception to this. We should expect this to be so between 0° C. and 4° C., but Amagat has shown that water is exceptional in this respect from 0° C. to 50° C.

Influence of Salt Solutions on Expansions.—In general, the expansion for a given rise in temperature is increased by the

presence of salts in solution. This holds for most liquids, including water. It is found that the lowering of the maximum density point is the same for all salts, if the same number of gram-equivalents of the salt is dissolved.

The Expansion of the Mercury Column of a Barometer.—In reading a barometer two errors are incurred with which we are now in a position to deal. The true mercury reading assumes the height to be at 0° C., a condition which rarely obtains in practice, and we must therefore correct for the expansion due to the temperature of the barometer column above 0° C. Further, the scale (usually brass) with which the instrument is fitted also expands, and as its true reading is also at 0° C. a correction for this too is necessary.

Let h_t denote the observed reading at $t°$ C.

Let h_0 denote the true reading at 0° C.

Had the scale not expanded, the reading would have been greater than h_t, *i.e.*, if β is the coefficient of linear expansion of the material of the scale, the reading at $t°$ C. would be $h_t(1+\beta t)$.

Now the pressure on the base of the barometric column is equal to the height × density × g, *i.e.*, using the usual notation for densities, and comparing the column at $t°$ C. with the equivalent column at 0° C., we have

$$h_0 g\rho_0 = h_t(1+\beta_t)g\rho_t.$$

But
$$\rho_t = \rho_0(1-\alpha_t),$$

where α is the coefficient of cubical expansion of mercury.

$$\therefore h_0\rho_0 = h_t(1+\beta_t)\rho_0(1-\alpha t),$$

and cancelling ρ_0, we get

$$h_0 = h_t(1+\beta t)(1-\alpha t)$$
$$= h_t\{1-(\alpha-\beta)t\}$$

Example 1.—*A barometer with a brass scale reads* 756 *mm. at* 20° *C. What would it read at* 0° *C. if the atmospheric pressure remained the same?*

Solution.—Let h_0 be the reading required.

Now $h_t = 756$, $t = 20°$ C.

The coefficient of linear expansion of brass = 0·00001888; and the coefficient of cubical expansion of mercury = 0·000182.

Since $h_o = h_t\{1-(\alpha-\beta)t\}$,

we have $h_o = 756(1-0{\cdot}0001632\times 20)$

$= 756(1-0{\cdot}003264)$

$= 756\times 0{\cdot}996736$

$= \mathbf{753{\cdot}53\ mm.}$

Expansion Thermostats.—A thermostat is an instrument which maintains a constant temperature, and one form of it makes use of the expansion of liquids on heating. This form of thermostat is frequently used in gas-ovens and incubators where constant temperature is required, and is so designed that when the temperature rises above the desired value, the gas-supply is diminished, but when the temperature gets too low, the gas-supply is increased so as to restore the oven or incubator to the original temperature. The action is therefore based upon the governor principle. It is illustrated in Fig. 38. The normal gas-supply is through E*d* and F on to the oven, but there is also a small hole *h* in the tube E*d* through which a small supply of gas may also pass direct to F. The upper part of the big bulb A contains paraffin, and the column BC is filled with mercury. The bulb A is in the oven, and acts as the detector of differences in temperature. Suppose the temperature rises above that which is required. The paraffin expands much more than the mercury, and the column BC is forced up, and closes the end *d* of the gas-tube E*d*. This automatically cuts off the gas supply except for the small quantity which is able to pass through the hole *h*, and which just suffices to keep the gas burning. If, however, the temperature falls, the paraffin, contracting more than the mercury, causes C to recede, and the gas-supply is increased.

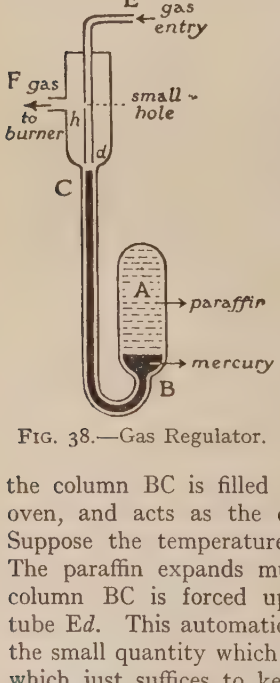

Fig. 38.—Gas Regulator.

EXAMPLES ON CHAPTER III

1. What do you understand by apparent expansion for a liquid? How is it connected with the absolute expansion of the liquid?
2. A flask holds 26·43 grams of liquid at 0° C., and 25·21 grams at 80° C. Find the apparent coefficient of expansion of the liquid in the flask.
3. Describe how to use a constant-volume air thermometer to measure the boiling-point of a liquid.
4. A thread of mercury occupies 100 divisions of a capillary tube at 0° C. How many divisions will it occupy at 100° C.? (Coefficient of linear expansion for glass =0·000007, and coefficient of cubical expansion for mercury =0·000183.) Illustrate from the above the importance, when taking the temperature of a hot liquid, of keeping the stem of the thermometer in the liquid.
5. Describe an experiment to determine how the volume of a mass of water changes with rise in temperature, and state the results obtained from such an experiment.
6. Describe an accurate method of determining the coefficient of absolute expansion of mercury.
7. What is a thermostat? Describe one which is suitable for an incubator.
8. Do you consider that water would be a good thermometric substance?
9. How would you find the coefficient of expansion of a given liquid by weighing a solid in it at different temperatures?
10. A weight thermometer weighs 104·5 grams empty, and contains 623·5 grams of mercury at 0° C. If the coefficient of apparent expansion of mercury =0·000154, find the temperature at which the total weight becomes 716·7 grams.
11. How can it be shown that water contracts when it is heated from 0° C. to 4° C., and that it attains its maximum density at the latter temperature? In what way are large masses of water, such as a lake or a pond, affected by this peculiar behaviour of water during a severe frost?
12. Describe how to use the weight thermometer for the measurement of temperature.
13. Describe how the coefficient of absolute expansion of mercury with rise in temperature may be found.
14. A thermometer which has been tested in the usual way is sunk to its 20° mark in a liquid, and reads 90° C. The mean temperature of the rest of the stem is 25° C. Find

the true temperature of the liquid, the coefficient of expansion of mercury in glass being 0·00015.

15. Give an account of the phenomenon of undercooling.

16. A portable oil-tank used for the retail distribution of illuminating oil holds 50,000 gallons at 60° F. It is delivered filled to a buyer on a very frosty morning at a temperature of 10° F. The mean coefficient of expansion of the oil relative to the tank is 0·0006 per degree F. By what volume does the tank appear to be short of being full when it is examined in the garage at 60° F ?

17. In an experiment to determine the coefficient of absolute expansion of mercury by Dulong and Petit's method, the following observations were taken :

Height of hot column = 76·58 cms.
,, of cold column = 76·32 cms.
Temperature of hot column = 18°·9 C.
,, of cold column = 0° C.

Calculate the coefficient of expansion of mercury.

18. What height of mercury column at 100° C. will balance a column 75·8 cms. at 0° C., given that the mean coefficient of absolute expansion of mercury is 0·00018 per degree C. ?

19. A cubical box whose side is 12 cms. is made of brass. It is filled with mercury at 14° C. The whole is heated to 100° C. How much mercury overflows, given that the coefficient of linear expansion of brass is 0·0000189 and the coefficient of cubical expansion of mercury is 0·000182 ?

CHAPTER IV

THE EXPANSION OF GASES

HAVING considered the expansion effect of heat upon solids and liquids, we now turn to the study of gases in relation to heat. This subject first received the serious attention of scientists early in the nineteenth century. It was soon recognised that the effect of heat on gases is influenced by the fact that both pressure and volume are easily changed. When dealing with solids and liquids, we were not concerned with pressure changes, since the influence of pressure is in such cases very small. In the case of a gas, however, the pressure, as well as the volume, is easily changed. To avoid confusion, therefore, we divide our experimental study of the influence of heat on gases into two parts. In the first, we consider the effects of changes in temperature upon the volume of a gas when its pressure is kept constant; and secondly, the influence of temperature changes upon the pressure, keeping the volume constant. Some idea of this need had existed during the eighteenth century, and a number of experimenters did in fact busy themselves with an attempt to work out the coefficient of expansion of air at constant pressure. The results of the different investigators were, however, so widely different from each other as to be useless, though Amontons, of whom we have already spoken in connection with thermometry, was not very far wrong.

Let us see how this investigation may be carried out nowadays.

Relation between Temperature and Volume at Constant Pressure.—A simple form of apparatus suitable for this experiment is shown in Fig. 39. A graduated capillary tube ABC closed at A is bent into a right angle at B, the bore along BC being wider than from A to B. The gas to be experimented upon is im-

prisoned in AD by a column of strong sulphuric acid which extends a few millimetres up the vertical tube. The tube is placed horizontally in a water-bath. The experiment is carried out practically at atmospheric pressure, the end C being open.

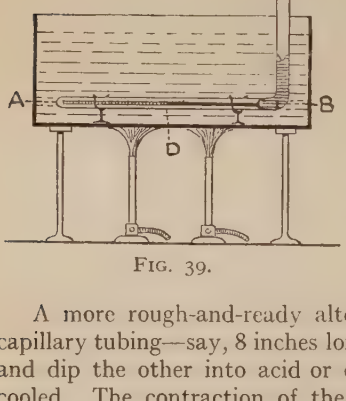

FIG. 39.

The cross-section being constant throughout the experiment, the volume of the gas is proportional to the length. The experiment consists in reading the length of the gas column for a series of temperatures. The bath should be well stirred to keep the temperature uniform.

A more rough-and-ready alternative is to take a length of capillary tubing—say, 8 inches long. Close one end by heating it, and dip the other into acid or even water before the tube has cooled. The contraction of the heated air will draw up some liquid—a half-inch is sufficient. Carefully withdraw the tube and hold it upright. The thread of liquid acts as a seal, and is drawn down until the temperature is normal, when the motion ceases and the entrapped air is at atmospheric pressure. Place the tube against a ruler and secure the arrangement with two or three elastic bands, and read the length of the imprisoned air column. Now repeat the experiment of Fig. 39 with this modified apparatus, but do not carry it out for a wide range of temperatures, especially if water is used as the seal. Why ?

Charles' Law.—The results of the experiments just described show that the relation between the volume and temperature of a gas when the pressure is constant is given by

$$v_t = v_o(1 + \alpha t) \quad . \quad . \quad . \quad \text{(i)}$$

This is consistent with the result for solids and liquids, but there is this important difference—**α does not differ from gas to gas ; it is practically the same for all gases,** and is equal to 0·00367. Expressed as a vulgar fraction, $\alpha = \frac{1}{273}$, which means that **all gases expand by an amount equal to $\frac{1}{273}$ of the volume at 0° C.**

for every degree rise in temperature. This important result is known as **Charles' Law.**

In the case of the Fahrenheit scale of temperatures, the expansion is $\frac{5}{9}$ of $\frac{1}{273}$, *i.e.* $\frac{1}{491\cdot4}$ of the volume at 32° F. for every degree Fahrenheit rise in temperature.

Jacques Alexandre César Charles (1746–1823), the first to deduce this important law, was a well-known experimenter of the late eighteenth and early nineteenth centuries. He was a professor of physics at the Conservatoire des Arts et Métiers, Paris; the very building, in fact, whose bulging walls were straightened by the force of contraction of iron bars as mentioned in Chapter II. Charles' interest in the behaviour of gases was practical as well as theoretical, and indeed he is best known as a balloonist and aeronaut. Hydrogen gas had previously been discovered (1766) by *Henry Cavendish* (1731–1810), and it was Charles who successfully suggested the use of this gas in 1783. Much about the same time another French scientist, *Joseph Louis Gay-Lussac* (1778–1850), was also experimenting upon gases. He was indeed the pioneer in the scientific investigation of the upper air, and with *Biot* made a famous ascent to an altitude of 13,000 feet for this purpose.

Gay-Lussac also discovered the law of expansion of gases at constant pressure, and published it in 1802, and for this reason the law is often spoken of as "Gay-Lussac's Law." Nevertheless, although Charles did not publish his results, he deserves full credit for the priority of discovery.

Relation between Temperature and Pressure at Constant Volume.—The relation between the temperature and pressure of a gas when the volume is kept constant is best determined by a slight modification of the apparatus just described. We have already referred to it in Chapter I. as a means for measuring temperature. A reference to Fig. 9 will at once show the principle of the method. The volume is maintained constant by keeping the mercury level at the horn-shaped datum-mark. When the expansion of the gas due to a rise in temperature forces the mercury down, more mercury is poured into the right-hand limb until the gas is again compressed to the original value. The difference in the two mercury levels is the excess of the pressure of the gas in the bulb above that of the atmosphere, and by varying the

temperature of the bulb a curve can be plotted connecting pressure and temperature.

An alternative device more frequently used for maintaining the level of the mercury in the left-hand limb at the index is that of the long flexible indiarubber connecting-tube as shown in Fig. 40. It has obvious advantages over the other form.

Results for Constant Volume—The Gas Equation.—Results show that when the volume of a gas is constant, the relation between pressure and temperature is given by

$$p_t = p_o(1 + \alpha t) \quad . \quad . \quad . \quad \text{(ii)}$$

α being the temperature coefficient of increase of pressure, α works out to 0·00367 for all gases, a result identical with that of the volume coefficient just considered. This remarkable result enables us to combine the two gas equations into the form

$$p_t v_t = p_o v_o(1 + \alpha t) \quad . \quad . \quad \text{(iii)}$$

for at constant volume $v_t = v_o$ and the equation reduces to equation (ii), whilst at constant pressure $p_t = p_o$, and the equation becomes as in (i).

We may rewrite (iii) thus :

$$p_t v_t = p_o v_o \alpha \left(\frac{1}{\alpha} + t\right)$$

But since $\alpha = 0{\cdot}00367 = \frac{1}{273}$

we have $p_t v_t = p_o v_o \alpha(273 + t)$,

and $p_o v_o$ being the standard condition of the gas at 0° C., and α being the constant, $p_o v_o \alpha$ will be a constant which we usually denote by R.

Hence we have $p_t v_t = R(273 + t)$ . . (iv)

Substituting T for $273 + t$, we have for any gas at pressure p and corresponding volume v the relation

$$\mathbf{pv = RT}. \quad . \quad . \quad . \quad \text{(v)}$$

This is spoken of as the **gas equation,** and gives the general relationship between pressure, volume, and temperature for any gas.

It will be clear to the student that in the case of Fahrenheit temperatures the above argument is the same, except that for $\frac{1}{273}$ in (iv) we substitute $\frac{1}{491}$.

Absolute Temperature.—In the gas equation $pv = RT$, T is equal to the ordinary temperature $t°$ C. plus 273, and is spoken of as the **absolute temperature.** The Centigrade temperature corresponding to 0° A. (*i.e.* 0° Absolute) is therefore 273° C., whilst the Fahrenheit temperature corresponding to 0° A. is $491 - 32 = 459°$ F.

What is the pressure corresponding to this temperature, if the volume is constant? Since $pv = RT$, if $T = 0$, pv must equal zero. But since the volume is finite and constant, it follows that $p = 0$. Hence at a temperature of $-273°$ C., provided that at this temperature the gas equation holds good, *the pressure of the gas would be zero.* We therefore speak of $-273°$ C. (or $-459°$ F.) as **the absolute zero of temperature.** In a constant volume gas thermometer, in which temperatures are read by referring them to the corresponding pressures obtained, it will be obvious that $-273°$ C. (or $-459°$ F.) is the lowest temperature which such a thermometer could record. We shall refer to this temperature again in the next chapter.

To convert a Centigrade temperature to the corresponding absolute temperature we add 273° to it, *e.g.* $18° \text{ C.} = 18 + 273 = 291°$ A., and $-15° \text{ C.} = -15 + 273 = 258°$ A., and so on.

To convert a Fahrenheit temperature to the corresponding absolute temperature, we add 459° to it, *e.g.* $60° \text{ F.} = 60 + 459 = 519°$ A., and $12° \text{ F.} = 12 + 459 = 471°$ A., and so on.

Some Worked Examples on the Gas Equation.—The equation $pv = RT$ may be written $\frac{pv}{T} = R = \text{a constant}$; *i.e.* for any gas, the product of its pressure and volume, divided by the absolute temperature, is a constant. The student will find it convenient, in dealing with questions involving the changing of a gas, to use the formula in the form

$$\frac{pv}{T} = \frac{p_1 v_1}{T_1}$$

i.e. $$\frac{\text{Old pressure} \times \text{old volume}}{\text{Old absolute temperature}} = \frac{\text{New pressure} \times \text{new volume}}{\text{New absolute temperature}}.$$

NOTE.—We refer to the conditions of a gas at 0° C. and 760 mm. pressure in the C.G.S. system (or 32° F. and 30 inches pressure in the British System) as **normal temperature and pressure,** and this is usually denoted by the symbols **N.T.P.**

Example 1.—*A certain quantity of gas occupies* 110 *cubic feet at N.T.P. What will be its temperature when the pressure is* 36 *inches and the volume is* 102 *cubic feet?*

Solution.—

$$\frac{\text{Old pressure} \times \text{old volume}}{\text{Old absolute temperature}} = \frac{\text{New pressure} \times \text{new volume}}{\text{New absolute temperature}}.$$

$$\therefore \quad \frac{30 \times 110}{(459+32)} = \frac{36 \times 102}{T_1}$$

$$\therefore \quad T = \frac{36 \times 102 \times 491}{30 \times 110}$$

$$= 546°{\cdot}4 \text{ A.}$$

$$\therefore \quad \text{Temperature of gas} = 546°{\cdot}4 - 459$$

$$= \mathbf{87°{\cdot}4\ F.}$$

Example 2.—*The density of oxygen at* 0° *C. under a pressure of* 760 *mm. of mercury is* 1·429 *grams per litre. A certain mass of the gas is enclosed in a cylinder whose volume is* 2·5 *litres under a pressure of* 780 *mm. at a temperature of* 12° *C. What is the mass of the gas in the cylinder?*

Solution.—We are given the density at N.T.P., and since the density $= \frac{\text{mass}}{\text{volume}}$, we can find the mass of gas in the cylinder if we find the equivalent volume at N.T.P.

We have—

Old volume = 2·5 litres.
Old pressure = 780 mm.
Old absolute temp. = 12 + 273

and

New volume = v_1.
New pressure = 760 mm.
New absolute temp. = 0 + 273

Hence, since $\frac{pv}{T} = \frac{p_1 v_1}{T_1}$

$$\frac{2{\cdot}5 \times 780}{285} = \frac{760 \times v_1}{273}$$

$$\therefore \quad v_1 = \frac{273 \times 2{\cdot}5 \times 780}{285 \times 760}$$

$$= 2{\cdot}46 \text{ litres.}$$

But density = 1·429 grams per litre

$$\therefore \quad \text{Mass of gas} = 1{\cdot}429 \times 2{\cdot}46$$

$$= \mathbf{3{\cdot}52\ grams.}$$

Example 3.—*A town is supplied with gas at a pressure of 3 inches of water above the atmospheric pressure. When the barometric pressure is 31 inches, and the gas temperature is 40° F., the town uses ten million cubic feet per week, and the company just pays its way. The next week the same mass of gas is used, but the average atmospheric pressure is 30 inches, and the average gas temperature is 50° F. Find how much gas the company must supply. Will it gain or lose on this week's work? (The specific gravity of mercury = 13·6.)*

Solution.—

Old pressure = 31 inches of mercury + 3 inches of water
= (31 × 13·6 + 3) = 424·6 inches of water.

Old volume = 10,000,000 cubic feet.

Old absolute temp. = 40 + 459 = 499° A,

and new pressure = 30 inches of mercury + 3 inches of water
= 411 inches.

New volume = v_1

and new absolute temp. = 50 + 459 = 509° A.

Hence $$\frac{424\cdot6 \times 10{,}000{,}000}{499} = \frac{411 \times v_1}{509}$$

$$\therefore \quad v_1 = \frac{509 \times 424\cdot6 \times 10{,}000{,}000}{499 \times 411}$$

= 10,537,903 cub. ft., say 10,538,000 cub. ft.

which is the amount of gas the company must supply at the same cost to itself as during the preceding week. It therefore makes a profit on the consumers' price of 538,000 cubic feet.

The Standard Constant Volume Thermometer. — We have seen that, owing to the uncertain behaviour of glass during the heating of a mercury-in-glass thermometer, errors are introduced in the readings intermediate between the two fixed points. Two mercury thermometers made of different kinds of glass will usually show slight differences in reading at almost any temperature. When we deal with gases, however, the expansion per degree rise in temperature is proportionately so much greater than in the case of liquids that the slight vagaries of the containing vessel affect the readings very little indeed. These expansions, too, are constant for a far greater range on both sides of the fixed points than is the case for liquids.

As a consequence, gases are far more suitable for standard purposes in thermometry than liquids. Fig. 40 shows a good form of constant volume gas thermometer of a standard type. The gas is contained in the bulb B, usually a litre cylinder of platino-iridium, and communicates with the reservoir AF by means of a fine steel tube. The mercury in the reservoir AF is kept at the level of the horn-shaped index.

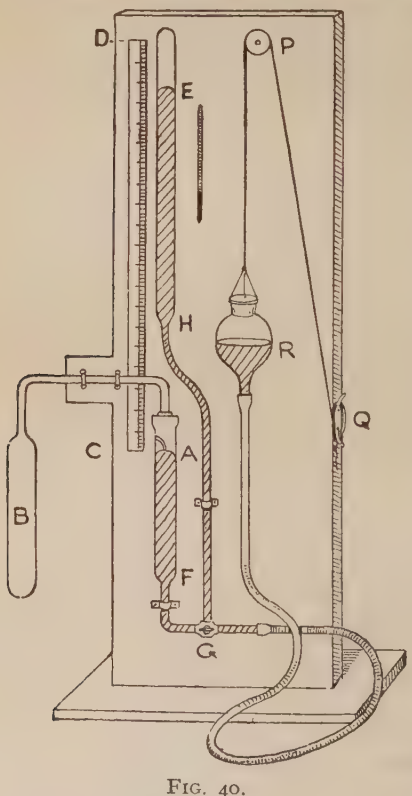

FIG. 40.

Communicating with AF by means of a three-way cock G is a long mercury column GH (bent so as to bring its closed widened extremity EH into a line with AF), and the reservoir R. This latter can be raised or lowered as desired by means of the cord PQ and pulley P, being connected to the horizontal tube at G by means of thick rubber tubing.

The apparatus is so arranged that when R is on the same horizontal level as A, the gas in B is at atmospheric pressure, and this is given by the height of E above A, as indicated on the scale CD. When the temperature of the gas in B alters, the mercury is maintained at the index A by adjusting the height of R, and this of course alters the height of the column EG. In every case, however, the excess of E above A as read on the scale gives directly the pressure in the bulb B, thus avoiding the

necessity for a separate barometer reading. The heights are carefully read with a cathetometer (*i.e.* a graduated telescope), and the temperature corrections are applied for the mercury and scale as indicated on page 49.

For ordinary purposes, the gas used is hydrogen, but at high temperatures this gas would leak through the bulb, and accordingly nitrogen (which does not leak in this way) is used for temperatures from about 500° C. to 1300° C. In 1815 *Dulong and Petit* made interesting comparisons of the mercury thermometer with the air thermometer, but more reliable comparisons were made later by *Henri Victor Regnault* (1810–1878). Regnault was a professor at the Collège de France until 1854, but he afterwards went into business as the director of a porcelain works at Sèvres. Although he contributed little to the theories of natural philosophy, he will always be remembered as a most painstaking and reliable experimenter, and his determinations of the various physical constants rank among the best of their kind in the history of physics.

Regnault found that the mercury and air thermometers are in close agreement between 0° C. and 100° C. (with the air thermometer about 2° C. below the other in the region of 50° C.), but that above 250° C. the mercury thermometer reads higher, the difference being 1° C. at 300° C. and 3° C. at 350° C. Later work has shown that the hydrogen thermometer is very reliable at very low temperatures.

Boyle's Law.—We have so far investigated two types of gas changes: (*a*) changes in pressure with changes in temperature, keeping the volume constant; and (*b*) changes in volume with changes in temperature, keeping the pressure constant. We saw that the results of these two types of change could be summed up by the gas equation

$$pv = RT,$$

where p and v denote the pressure and volume respectively, T the absolute temperature, and R a constant.

We now proceed to a third type of change—(*c*) the relation between pressure and volume when the temperature is kept constant.

This relationship at once follows from the gas equation, for

if T is constant, RT is also constant, and, if we denote its value by K, we get

$$pv = K,$$

i.e. **the product of the pressure and the volume of a gas is always constant, provided the temperature remains unaltered.** This important result is known as **Boyle's Law.** It was discovered by Robert Boyle in the year 1661.

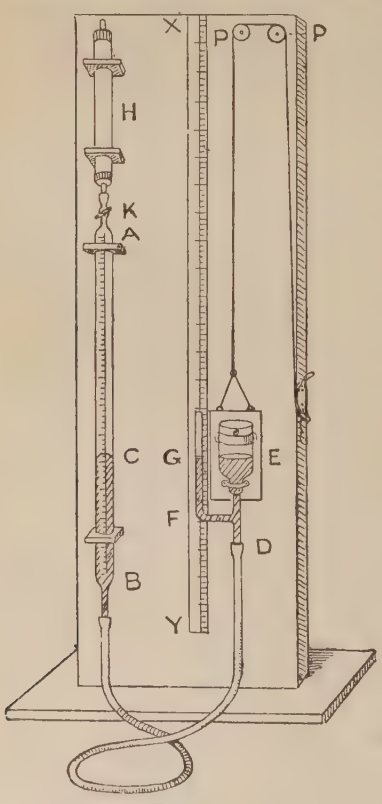

FIG. 41.

The experimental verification of this law is illustrated in Fig. 41. The gas to be tested is introduced into a graduated burette AB by means of the stop-cock K, after having been carefully dried by being passed through the calcium-chloride tube H. The gas is trapped by the mercury column CB communicating with the mercury reservoir E by means of a long piece of thick rubber tubing. To facilitate the reading of the mercury level in E, a narrow side tube FG communicates with it as shown. FG is immediately in front of the long scale XY, over which it can be made to slide—by drawing a cord over the pulleys P at the top of the stand. The procedure of the experiment is as follows. The index-tube FG is adjusted until the levels in C and G are the same. The pressure in the reservoir E being that of the atmosphere, the pressure of the gas in AC will also be atmo-

spheric. The volume of the gas is the product of the height AC and the cross-sectional area of the tube. But throughout the experiment the cross-section of the tube is unaltered, and therefore the volume of the gas may always be represented by the height of AC. This is accordingly read on the burette-scale.

We now raise the reservoir E by means of the cord and pulleys to some position E′ (Fig. 42). The mercury level rises from C to C′, but this is not as great as for the right-hand limb. We have increased the pressure exerted against the gas, which has in consequence diminished in volume. The increase in pressure is given by *h*, the difference in levels between G′ and C′, and this is accordingly added to the atmospheric pressure (previously read on a barometer) to give the new pressure of the gas. Also the volume A′C′ is read off, and the level of the reservoir is again raised for a third reading. The pressure in this way is increased in steps until several readings have been obtained for pressures greater than that of the atmosphere. The reservoir is then again brought back to its initial position, fresh readings are taken at pressures less than atmospheric, *i.e.* the reservoir E is lowered in successive steps instead of raised, and the difference of levels *h* is now subtracted from the barometric pressure to give the pressure in AC.

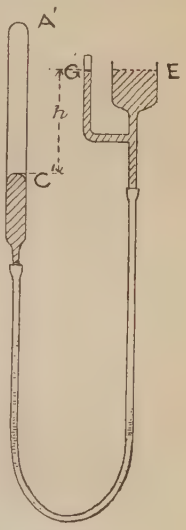

FIG. 42.

In each case the pressure is multiplied by its corresponding volume, and it will be found within the limits of experimental error that the product is constant, thus verifying Boyle's Law.

Illustrative Examples on Boyle's Law

Example 1.—*There is a pressure of* 72 *cms. of mercury on* 800 *c.c. of gas. What pressure must be applied to reduce the volume to* 650 *c.c.? There is no change in temperature.*

Solution.—

Old pressure × old volume = new pressure × new volume

$$\therefore \quad 72 \times 800 = p_1 \times 650$$

$$\therefore \quad p_1 = \frac{72 \times 800}{650} = \mathbf{88{\cdot}6 \text{ cms. of mercury.}}$$

Example 2.—*If the cylinder of an air-pump is one-quarter of the size of the receiver, what fraction of the original air will be left after 5 strokes, and what will be the actual pressure in the receiver if the atmosphere is at 76 cms. ?*

Solution.—Call the volume of the cylinder 1 part.
Call the volume of the receiver 4 parts.

After one stroke, the air that originally filled 4 parts now occupies 5 parts.

∴ Four-fifths of the original air is left in the receiver.

Similarly $\frac{4}{5}$ of $\frac{4}{5}$ air is left in the receiver after the second stroke.

∴ Fraction of original air left after 5 strokes
$= \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = 0{\cdot}8^5 = 0{\cdot}33.$

And pressure in receiver $= 0{\cdot}33 \times 76 = \mathbf{25{\cdot}3 \text{ cms.}}$

Example 3.—*A cylindrical diving bell is 8 feet high. It is lowered till the top is 15 feet under sea-level. The specific gravity of the sea water is 1·03, and the barometer reads 30·8 inches. How much does the water rise within the bell ?*

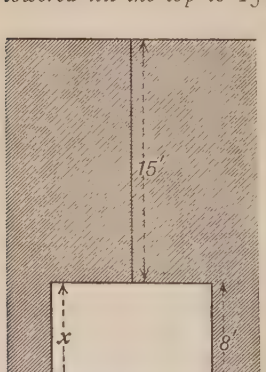

FIG. 43.

Solution.—We apply Boyle's Law to the contents of the bell before and after immersion (Fig. 43).

I.e. old volume = 8 units (since the volume is proportional to the height, the cross-section remaining constant),
and old pressure = 30·8 inches.
Let new volume = x units.
New pressure = 30·8 inches of mercury $(15 + x)$ feet of salt water

$$= 30{\cdot}8 + \frac{(15+x)\times 12\times 1{\cdot}03}{13{\cdot}6} \text{ inches of mercury}$$

$$= 44{\cdot}5 + \frac{12{\cdot}36}{13{\cdot}6} \text{ inches of mercury.}$$

Hence, by Boyle's Law

$$8\times 30{\cdot}8 = x\times\left(44{\cdot}5+\frac{12{\cdot}36}{13{\cdot}6}x\right),$$

which gives a quadratic equation from which x may be calculated.

The Story of Boyle's Law.—It may be interesting to recount briefly the circumstances under which the Hon. Robert Boyle discovered the law named in his honour. He had long been interested in the subject of air pressure, and in 1660 he published a work called *The New Experiments . . . touching the Spring in the Air*. He illustrated the pressure exerted by the air by calling to his aid a sort of analogy in which he pictured the air particles as behaving as though each was like a spherical spring. The effect of pressure was to compress the spring and so to diminish the volume. This book met with some criticism, particularly from a certain Jesuit father, Franciscus Linus, of Liège, who objected that the air could not perform "such great matters as the counterpoising of a mercurial cylinder of 29 inches," and he not only declared that the mercury hangs by invisible threads from the upper end of the tube, but also asserted that he actually felt the threads when he closed the upper end with his finger. It was in reply to this farrago of nonsense that Boyle began those further experiments (a crude form of what has already been described above) that resulted in the establishment of his famous law.

In France, however, the law is not known as Boyle's Law. Fourteen years after Boyle's famous experiment, *Edmé Mariotte* (1620–1684), a French scientist of repute, published a book, *Sur la nature de l'air*, in which he described his own independent discovery of the law "that air condenses in proportion to the weight by which it is loaded." It seems indeed curious that Mariotte should have been unaware of Boyle's researches, yet so it was, and in all French schools to-day they speak of Mariotte's Law just as we do of Boyle's Law.

Isothermals.—If we plot a graph showing the pressures of a gas as ordinates, and the corresponding volumes at constant temperature as abscissæ, we get a curve which gives us the graphical representation of Boyle's Law. Such a curve is called

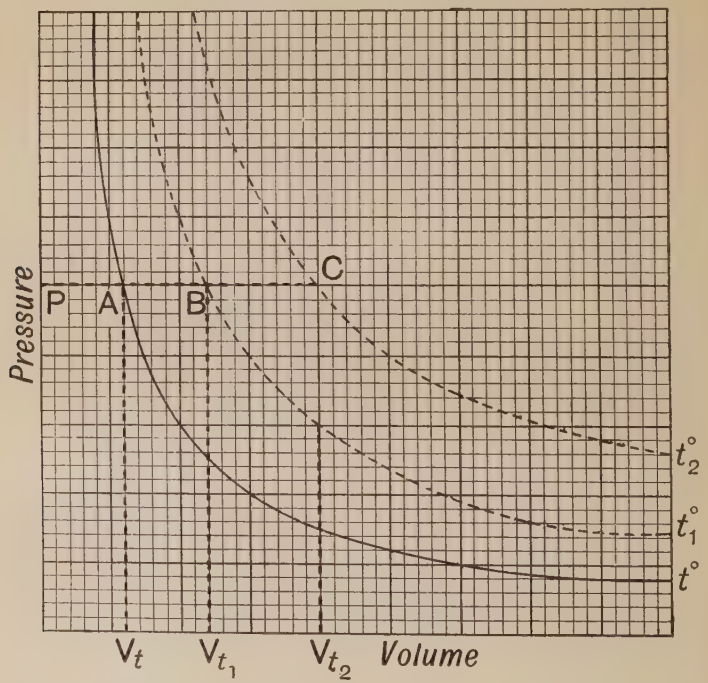

FIG. 44.

an **isothermal,** and we shall have a different isothermal for every one temperature.

Fig. 44 shows the isothermals for three temperatures, $t°$ C., $t_1°$ C., and $t_2°$ C. The equation to an isothermal being $pv = K$, K will have a different value in each case. From the equation it will be seen that the curve is a portion of a rectangular hyperbola.

It will be evident, moreover, that in the figure, $t_1°$ C. will be greater than $t°$ C., and $t_2°$ C. will be greater than $t_1°$ C., for if we consider for a moment the state of the gas represented by the point A on the $t°$ C. isothermal, its volume is given by v_t in the figure. If we now raise its temperature to $t_1°$ C., but keep its pressure constant, the gas expands, and its volume increases from v_t to v_{t1}. To represent it on a graph, therefore, we must move in a horizontal line through A to the right. The pressure is thus unaltered, and we get to the point B. Similarly, a further increase in temperature to t_2 will bring the gas still farther to the right to the point C on the $t_2°$ C. isothermal. Thus in successive isothermals the temperatures increase to the right, and decrease to the left.

Perfect Gas.—No gas completely obeys Boyle's Law, which, like Charles' Law, is only approximately true. When it was first seen that the gas equation was not completely obeyed, it became convenient on theoretical grounds to refer to a *perfect gas* as being one which completely obeys the Gas Laws, and this term is a very useful one. No gas is perfect in this sense : in the case of hydrogen it was found that within certain limits the deviations were of an opposite character from that of other gases, and this was accordingly spoken of as being " more than perfect."

EXAMPLES ON CHAPTER IV

1. State Boyle's Law and Charles' Law, and from these laws show that $pv = RT$. What do R and T symbolise ?
2. A sample of gas was found to have a volume of 100 c.c. at 18° C. and 72 cms. of mercury pressure. Calculate at what temperature it would have a volume of 400 c.c. and a pressure of 100 cms.
3. State exactly what you understand by the absolute zero of temperature.
4. A flask is heated from 15° C. to 100° C. What fraction of the air which is in the flask at the beginning of the experiment leaves it during the heating ?
5. Describe the construction of a gas thermometer. What are the advantages and disadvantages of the gas thermometer compared with other thermometers ?
6. Describe a form of air thermometer, and explain what measurements you would make to determine by it the

temperature of a vessel of hot water. Give a brief account of the corrections to be applied.

7. Some air is in the space above the mercury in a barometer. The mercury stands at 29 inches in the tube, the air space above being 4 inches long. The tube is now pushed down into the cistern so that the space above the mercury is only 2 inches long, and now the mercury stands at 28 inches. At what height would it stand in a perfect barometer ?
8. Find the number of cubic feet in a steel bottle intended to hold, at 120 atmospheres pressure, when the temperature is 25° C., 20 cubic feet of oxygen under normal conditions.
9. A certain quantity of gas occupies 200 litres at 120° C. and 750 mm. pressure. What volume will it occupy at a pressure of 5 atmospheres and 1444° C. ?
10. Air at atmospheric pressure (32 feet water barometer) is taken down to 30 feet under water, and liberated to form a spherical bubble. Show that the diameter of this increases approximately by one-fourth when it reaches the surface.
11. A cylindrical diving bell is 6 feet high, and its top is 10 feet under water. To what height has the water risen in the bell if the mercury barometer is at 30·5 inches ?
12. A balloon of capacity 60,000 cubic feet is to be inflated with hydrogen from cylinders containing 4 cubic feet of gas at a pressure of 1500 lb. per square inch. How many such cylinders will be required ? Assume that the temperature of the gas remains unaltered, and that the pressure inside the envelope is to be 15 lb. to the square inch.
13. At a given instant the gas in a cylinder occupies 6 cubic feet at a pressure of 15 lb. per square inch, and the temperature is 20° C. At another instant the same quantity of gas is compressed to 150 lb. per square inch, and the volume is 2·5 cubic feet. What is the new temperature of the gas ?
14. A balloon of capacity 10,000 cubic feet contains 8000 cubic feet of hydrogen when on the ground. Explain what will happen if the balloon rises steadily to a height where the new pressure is 10 lb. per square inch, if the pressure on the ground is 14 lb. per square inch. When will this balloon cease to rise ?
15. Until recently, illuminating gas was paid for by volume. By this method will a consumer receive a greater mass of gas per 1000 cubic feet (*a*) when the barometric pressure is high or when it is low, (*b*) when the temperature is high or when it is low ?
16. A caisson (*i.e.* a sort of large-size diving-bell) is lowered into a lake until the water in it is 700 cms. below the surface of the lake. Find the density of the air in the caisson if

it is 0·0012 gram per c.c. on the surface, the barometer reading 76 cms.

17. A motor tyre is pumped up at a pressure of 80 lb. per square inch when the air is at 63° F. The car is driven until the air within becomes 140° F. (*a*) Assuming that the tyre does not stretch, find the air-pressure. (*b*) What will be the pressure if the tyre stretches one-twentieth of the original volume ?

18. A steel cylinder contains 15 cubic feet of oxygen at a pressure of 3000 lb. per square inch at a temperature of 20° C. If 1 cubic foot of oxygen at 0° C. and a pressure of 15 lb. per square inch weighs 0·089 lb., what is the weight of gas contained in the cylinder ?

19. Determine the value of R in the equation $pv = RT$, given that 1 cubic foot of nitrogen at 0° C. and a pressure of 15 lb. per square inch weighs 0·078 lb.

20. At one instant a cylinder contains 40 cubic inches of gas at a pressure of 13 lb. per square inch and a temperature of 100° C. At another instant the volume is reduced to 8 cubic inches and the pressure is 120 lb. per square inch. What is the temperature ?

21. Some gas occupies a volume of 91 c.c. at N.T.P. At what temperature will it occupy 100 c.c. if its pressure be reduced to 730 mm. ?

22. The coefficient of expansion of air was approximately estimated by heating up an open flask of 34·2 cubic inches volume until the temperature of the air within was 206°·6 F. It was then inverted in a water-bath whose temperature was 57°·2 F. with the open end under water. When the whole had cooled, 7·6 cubic inches of water had entered the flask. What was the coefficient of expansion of the air ?

23. After 75 c.c. of gas have been collected in a tube over mercury, the level of the mercury is found to be 12 cms. above the level of the mercury in the reservoir. The barometer reads 770 cms. and the temperature is 15° C. Find the volume of the gas at N.T.P.

CHAPTER V

THE KINETIC THEORY OF MATTER

Molecular Structure of Matter.—The Kinetic Theory is based on the molecular structure of matter.

All substances consist of an immense number of very tiny particles, called molecules, which are not actually in contact, but are separated from each other by minute distances which differ in the different states of matter.

We can best conceive the idea of a molecule by imagining a small portion of a substance, say water, to be divided up and subdivided again and again. Actually this continued subdivision is impracticable owing to our inability to cope with the extremely minute portions produced, but for theoretical considerations we may assume that such practical difficulties can be overcome. For a long time the process of subdivision merely results in increasingly small particles of water, but eventually a limit is reached at which a further subdivision produces not two particles of water, but one of hydrogen and one of oxygen, *i.e.* two substances whose chemical properties are different from the original.

In other words, we have reached the smallest particle of water which can exist as such, and we speak of this as a *molecule* of water. *A molecule therefore is the smallest portion of a substance which can exist in a free state and retain the properties of that substance.* It will be seen from the example quoted above that a molecule *is* capable of further subdivision: water molecules can be split up into molecules of hydrogen and oyxgen, whilst carbon dioxide molecules are built up of molecules of carbon and oxygen. When we consider an element, however, as distinct from a chemical compound, the divisions we obtain are called *atoms*. Thus a molecule of phosphorus consists of four atoms, three or five being an impossible number; a molecule of hydrogen consists of two

atoms, whilst a molecule of zinc has only one atom. In monatomic substances such as zinc the terms molecule and atom are synonymous.

We may therefore define an atom as being the smallest particle of matter which can take part in a chemical change.

Electrons.—Recent physical research has shown that atoms are not the final stage of subdivision of matter. According to the Electron Theory which has been developed by Sir J. J. Thomson and others, and which is now generally accepted by all scientists, the atom in its normal state consists of a comparatively large nucleus to which is attached a positive charge of electricity, surrounded by a number of much more minute particles to each of which is attached a negative charge of electricity. The nucleus is spoken of as the positive electron, and the surrounding particles are negative electrons. These latter are about one two-thousandth part of the mass of the positive electron. They are not at rest, but revolve around the nucleus at varying distances in closed orbits like a miniature solar system.

From an electrical point of view, in the normal state the negative charges counterbalance the central positive charge; but if, as will occasionally happen, one of the negative electrons is "lost" to a neighbouring molecule, there is a preponderance of positive electricity in the atom, which we express by saying that it is positively charged. Similarly, when an additional electron is "caught" from a neighbouring molecule, there is a preponderance of negative electricity, and we say the atom is negatively charged.

The Electron Theory has had a great influence on modern physical thought, but from the point of view of this book we are chiefly concerned with it from the fact that it has served fully to confirm the view as to the molecular structure of matter.

Relative Molecular Magnitudes for Solids, Liquids, and Gases.—We have no certain knowledge as to the shape or size of molecules. For general purposes we may talk of them as though they are spherical in shape, but it should be understood that this is only a statement of convenience which cannot be true. Estimates as to the approximate "diameter" of a molecule agree in a value of the order 2×10^{-8} cm., a magnitude far too small for adequate human conception. Some very inadequate idea of this may be attempted from the following illustration: if a drop of

water be magnified to the size of the earth, the molecules in it would be of the size of footballs.

From the point of view of the Molecular Theory, the chief distinction between solids, liquids, and gases is in the difference between the average distances apart of the molecules of which they are formed. In solids they are closest together, in liquids they are not so close together, and in gases they are farthest apart. If we denote by d the diameter of a molecule, and by x the average distance from centre to centre, then in solids x is of the order $1{\cdot}5d$ to $3d$, in liquids $2d$ to $4d$, whilst in gases x is of the order $10d$ to $15d$. As an illustration of the difference in this respect between liquids and gases, we know that 1 c.c. of water occupies a volume of 1700 c.c. when it is converted into steam by boiling, so that for steam the average distance between molecules is $\sqrt[3]{1700}$ times the distance for water, *i.e.* about twelve times the distance apart for water.

The first reliable determination of the number of molecules present in 1 c.c. of a gas at normal temperature and pressure was made by Loschmidtz, who deduced a value of 10^{20}. More recent determinations show this to be too large, a later value being $2{\cdot}7 \times 10^{19}$.

The Kinetic Theory of Gases.—According to the Kinetic Theory, the molecules of a gas are not at rest with respect to each other, but are in a state of rapid motion, such motion being in straight lines, and in all directions. There is no system in the movements, which are consequently referred to as non-directed or haphazard motion. The motion of a molecule will be constantly interrupted by collision with other molecules. The collisions will not necessarily be direct, but they will produce rebounds and a rearrangement of velocities both in magnitude and direction. The velocities will therefore vary within limits.

The average distance which a molecule is able to traverse without interruption is known as the *mean free path* in the gas. The mean free path is different for different gases, and alters with alteration of temperature. It follows from this view of the behaviour of the molecules that they possess mechanical energy. This energy is not manifest in the body of the gas as a whole; the gas may, for example, be enclosed in a box and outwardly be quite still, yet inwardly, from the point of view of the molecules

themselves, the haphazard motion which they possess must mean the existence of a large amount of kinetic energy, and we speak of the total of this for all the molecules as the *internal energy of the gas*.

The Influence of Temperature.—The motion of translation is not the only one to which the molecules of a gas are subjected; there is also a rotary motion about the centre of mass, as would be produced, for example, by a "glancing" collision with another molecule. The motions of the individual atoms within the molecules, and independent of the molecule as a whole, should also not be forgotten; but the motion of translation is easily the most important factor.

Temperature has a great influence on the velocity of the molecule. We may express the result by saying that *the kinetic energy of the molecule is proportional to the absolute temperature of the gas*. If u is the mean velocity of a molecule, then $u \propto \sqrt{T}$. Hence the effect of increase in temperature is to make the molecules move faster, and so to increase their kinetic energy.

Evidence of Molecular Motions in Gases.—That the molecules of a gas must be in motion, even when confined in a "still" space, is evident from a few simple everyday observations. For example, if a gas like ammonia, having a strong smell, be introduced at one end of a room, in a very short time we are conscious of its presence in every part of the room. Again, suppose we have two spherical vessels A and B connected as shown in Fig. 45, the upper one containing, say, hydrogen, and the lower a much heavier gas like carbon dioxide. On opening the stop-cocks connecting the two we find after some hours that both gases fill both spheres exactly in the same proportions. This phenomenon is spoken of as *diffusion*, and clearly demonstrates the existence of molecular motions.

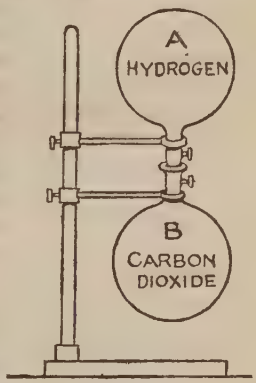

FIG. 45.

The Pressure of a Gas.—The well-known fact that gases have the property of indefinite expansibility at once follows from the

Kinetic Theory. If the walls of a vessel filled with gas are suddenly withdrawn so that the gas is set free in a much bigger space, the molecules previously in collision with the walls, or moving towards them, are no longer impeded, *i.e.* their further motion along the line of their velocity is not interfered with. Consequently they are able to move on, thus giving them more room, and increasing the mean free path of the molecules in the centre of the space. The molecules thus adjust themselves to the new space conditions, and the amount of their expansion is only limited by the walls of the containing vessel. Of course the molecules hamper each other in collision, and therefore the adjustment should not be instantaneous, but should take some little time. Practically, however, no appreciable time can be observed. Thus, if we have a gas confined in a cylinder by a movable piston, and the piston is suddenly withdrawn, however sudden the movement, the molecules of the gas appear to follow it and take up the new volume instantaneously. This can only be explained by the fact that the mean velocity of the molecules must be very high, as indeed it is. At normal temperature and pressure, the velocity of a molecule is of the order of from one and a half to twice the velocity of sound; in air, for example, the molecular velocity is about 500 metres per second.

These high velocities manifest themselves externally by what we call the **pressure** of the gas. The limit to the molecular motions is determined by the walls of the containing vessel; these are constantly bombarded by the molecules in their tendency to expand farther, and we speak of this bombardment as the pressure which the gas exerts upon the walls of the vessel.

Influence of Molecular Attraction.—It should be noticed that this behaviour of a gas is due initially to the fact that the average distance apart of the molecules from each other is large in comparison with their diameters. Were this not so, we should have to consider as an important factor the gravitational attraction of the molecules towards each other. This would act as a "binding" influence, and would oppose the freedom of movement and the tendency to expansion, but in the case of gases this influence is relatively small and does not interfere materially with the property of unlimited expansibility.

Extension of the Kinetic Theory to Liquids and Solids.—Molecular motions undoubtedly exist in liquids as well as in gases. For example, a small layer of liquid left exposed to the air gradually disappears. This can only be due to the slow but persistent loss of molecules to the air. The molecules of a liquid are in constant collision with each other, but occasionally one will be moving upwards, and it will therefore penetrate into the air above.

Compared with gases, the mean free path for liquids is small, and the influence of molecular attraction now becomes an important factor. As a consequence, while the molecules are free enough to enable one portion of liquid to "slip over" another portion, *i.e.* to be poured out, and to permit the liquid immediately to assume the shape of the containing vessel, yet it is unable to expand, and so retains a fairly constant volume. Within the liquid itself, however, a molecule is able to travel by a series of collisions from one part of the liquid to another: its position may constantly change.

In the case of solids the mean free path is very small, and the molecular attractions are relatively so large that freedom of motion cannot exist, and the molecules are "bound" to mean positions from which they can only move a very little distance. Accordingly, the relative positions of the molecules of a solid remain practically unaltered, and it therefore preserves its shape and volume. The cohesive forces which bind the molecules of a solid together are surprisingly great, and to overcome them by separating the molecules requires an immense force of tension.

That molecular motions do definitely exist in solids has been demonstrated by the following experiment. A block of pure silver was fitted exactly on a block of pure gold and left undisturbed for a few years. On examination, each block was found to possess a very thin surface of alloy of gold and silver. This experiment has been successfully repeated for many other metals, and is particularly successful at high temperatures.

Deduction of Boyle's Law from the Kinetic Theory.—Suppose we have gas filling a cube the length of whose side is a. The effect of the collision of the molecules with the sides of the cube is to produce a pressure. We may regard the motion of the molecules at any instant as equivalent to equal amounts of

movement in three directions—to and fro, left and right, and up and down. Of course most molecules are really moving slantwise, but as each slantwise movement is partly left and right, partly to and fro, and partly up and down, the assumption is justified.

Let n = the total number of molecules.

Let $\frac{n}{3}$ = the total number of molecules moving in one of the three possible directions.

Now suppose the size of the cube to be diminished to $\frac{a}{2}$.

The number of molecules of course remains n.

The pressure on a face is the force per unit area on that face.

As before, $\frac{n}{3}$ molecules move to or from one face at any instant, but the area of this face is only one-quarter of what it was before, so that in respect of area alone four times the number of blows will be struck on the face than before in the same time. But again, the molecule has only half the former distance to go before striking the opposite face, so that in this respect the number of blows per unit time is also doubled. Combining these two factors, we see that when the length of the cube is halved the pressure is increased eightfold. But also the volume becomes one-eighth of the former volume, so that the new product of pressure and volume is the same as the old product.

It is left as a simple exercise to the student to work out the more general case when the edge of the cube is diminished to $\frac{a}{N}$ of the former size.

CHAPTER VI

CHANGE OF STATE—SOLID TO LIQUID

Change of State.—We have seen in the Kinetic Theory that the chief differences between the three states of matter arise from the fact that if d is the average diameter of a molecule, the average distance between neighbouring molecules in solids is of the order $1{\cdot}5d$ to $3d$, in liquids it is of the order $2d$ to $4d$, whilst in gases it is $10d$. to $15d$.

We have also seen that when heat is applied to any substance it *expands, i.e.* its mean free path increases. Hence, by a steady application of heat to a solid, it is reasonable to expect that its molecules in time become sufficiently separated by expansion to convert it into a liquid, and similarly the liquid will eventually be converted into a gas. This does in fact take place.

Solid to Liquid.—If we take a crystalline solid, and increase its temperature by the application of heat, the rise in temperature is accompanied by an increase in volume. At a particular stage in the operation, the body undergoes a physical change of state, and the crystalline solid begins to be converted into a liquid, *i.e.* it begins to melt. As soon as this melting process has commenced, *the supply of heat produces no further increase in the temperature of the body,* and the temperature will remain constant so long as the body is kept well stirred and there is still some of the solid left to be melted.

The heat energy which before was being used up in increasing the temperature now all goes to convert the solid into the liquid state.

As soon as the last trace of the solid has disappeared, provided we continue to supply heat to the liquid, the temperature once again begins to rise.

The temperature which remains steady during the continued

application of heat is spoken of as **the temperature of the melting of the solid,** or, more shortly, the **melting-point of the solid.**

Liquid to Solid.—If after melting we abstract heat from the body, we find that the above process is reversed. The temperature at first begins to fall, but at a certain stage in the operation the cooling ceases to be accompanied by a decrease in temperature. Instead, some of the liquid begins to be converted into a solid, *i.e.* it *freezes* or *solidifies*, and *the temperature during the freezing process remains constant* so long as the mixture of liquid and solid is kept well stirred. This temperature is called the **freezing-point of the liquid,** and for crystalline substances it is identical with the melting-point of the solid.

As soon as the last trace of liquid has disappeared, further cooling again results in a decrease in temperature.

Latent Heat.—The amount of heat which is supplied to or abstracted from a substance during the process of a change of state is spoken of as **latent heat.** Before the days of Joule, when the significance of heat as a form of energy was not realised, it was merely considered that when a substance melts, since the temperature has ceased to rise, the heat has disappeared, but that it reappears when the melting is completed, the temperature again going up. The term *latent* or *hidden heat* was then undoubtedly appropriate, but now that we know that the heat does not disappear, but merely changes its function from raising the temperature to that of changing the state of the body, the name is no longer appropriate ; it is nevertheless retained.

The phenomenon of latent heat was first observed and explained by *Joseph Black* (1728–1799), a Glasgow University professor of chemistry who was born in France of Irish parentage. It was in 1756 that Black began to wonder at the slowness with which ice melts and water boils, and it was he who referred to the heat that was causing this without producing a rise in temperature as " latent." Black never published his results, though he referred to them in his lectures. He was nevertheless a great pioneer in both the qualitative and the quantitative study of heat, and his work on latent heat had enormous practical influence through his friendship with James Watt, who, largely with Black's inspiration and sympathy, so greatly improved the steam-engine.

The student should remember that **no change of state is possible without an absorption or an emission of heat.** Thus, when a substance melts, heat is absorbed. This heat may or may not be artificially supplied, as by a Bunsen burner; if it is not so supplied, then heat is abstracted from the surroundings, which accordingly become cooled. We shall meet with several examples of this when we deal with changes from liquid to vapour in the next chapter. The converse is also true. When a body freezes, it gives out heat; and only by this abstraction of heat can the freezing be made to occur. The heat thus abstracted goes to the surroundings, which are accordingly warmed. This fact has been utilised during cold weather in vegetable stores. Vegetables "freeze" at a temperature slightly below 0° C. To prevent them from thus spoiling, tubs of water are placed among them. The water freezes at 0° C., and in doing so evolves heat sufficient to prevent the room from reaching the temperature at which vegetables "freeze."

Another example of the influence of latent heat is seen in the case of the supercooling referred to in Chapter III. It will be remembered that, under favourable conditions, it is possible to cool water to a temperature of $-5°$ C. or $-6°$ C. without freezing. If this supercooled liquid be suddenly disturbed, as by stirring with a thermometer, the whole mass at once solidifies, and the temperature goes up to 0° C. The reason for the rise in temperature is that the freezing is accompanied by a liberation of heat, and it is this heat which produces the rise.

Case of Amorphous Substances.—The case of non-crystalline substances is interesting in that there is no definite melting-point. Instead of a well-defined and abrupt transition from the solid to the liquid state, there is a softening stage during which the substance passes through all the gradations of viscosity. The temperature too is not constant, but continually rises or falls, as the case may be, although during the change of state the change in temperature is very slow indeed. There is, therefore, not a melting- or freezing-point for amorphous substances, but a melting or freezing-region.

Figs. 46 and 47 show the types of temperature curves for crystalline and non-crystalline substances respectively. While the change of state is clearly defined in the former, it is very

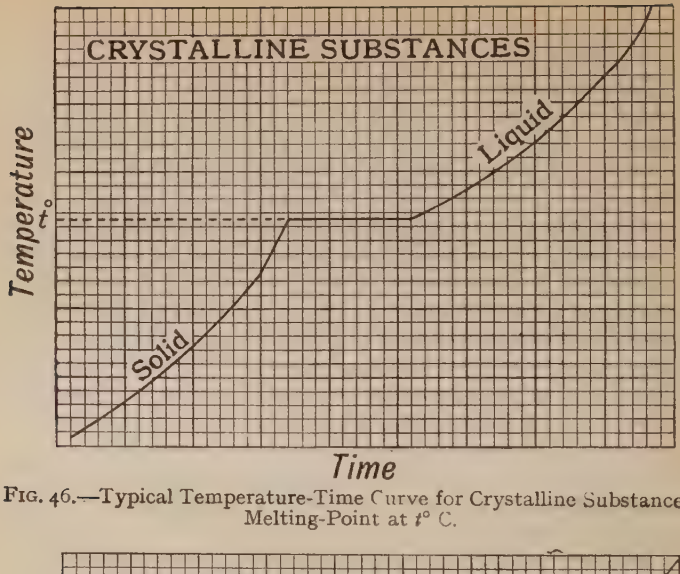

FIG. 46.—Typical Temperature-Time Curve for Crystalline Substances: Melting-Point at $t°$ C.

FIG. 47.—Typical Temperature-Time Curve for Amorphous Substances: Melting Region, $t_1°$ C. to $t_2°$ C.

vaguely defined in the latter. Examples of amorphous substances are glass, tar, wax, resin, guttapercha, alcohol, carbon, etc. The softening stage is taken advantage of in the case of glass in the making of scientific apparatus, etc.

Change of State and Volume.—The change in the physical state is generally accompanied by a change in the volume. On reaching the melting-point, the body suddenly expands much more than it has been doing. This abnormal expansion continues throughout the melting process, but afterwards in the liquid state it again becomes normal.

Ice is an important exception to the general rule. *Instead of an expansion, there is a contraction.* When the ice has all been converted to a liquid, the contraction continues at a much slower rate until 4° C. is reached, after which, as we have already seen, it begins to expand. It is interesting to note that in Robert Boyle's book, *New Experiments and Observations touching Cold,* published in 1665, one of his experiments was to determine the relative volumes of ice and water. He used for this purpose what was then spoken of as a "philosophical egg," *i.e.* a tube terminating in a bulb, the stem of which was graduated in terms of the volume of the bulb. Into this he poured water and then immersed the bulb in a freezing mixture (allowing the stem to project above the freezing mixture in order to avoid bursting by allowing for expansion—a shrewd appreciation of the difficulties of the experiment). He then measured the final position in the stem after freezing had taken place. The result—uncorrected for changes in the glass—gave an expansion of about 11 per cent.—rather a high value.

Iron is another exceptional substance in this respect. *Wrightson* has found that molten cast-iron expands some 6 per cent. on solidifying. His experiment consisted in holding a cast-iron ball suspended from a spring balance in a bath of the same metal in a molten condition. The solid ball being heavier (of density 6·95 before immersion as compared with 6·88 for the same liquid metal), it was completely immersed below the liquid level. As its temperature went up to the region of melting, it floated on the surface, indicating a density of 6·5; it was in a very soft condition, and later suddenly assumed the liquid state. Only substances which expand on solidifying, or at least which do not

appreciably contract, are suitable for the making of castings; if a substance contracts, it cannot retain the shape of the mould. This is why gold and silver coins are not cast, but stamped. Iron makes a good casting, as does also of necessity printing type metal, usually an alloy of copper, antimony, and lead.

Figs. 48, 49, and 50 show typical temperature-volume curves for crystalline substances, ice, and amorphous substances respectively.

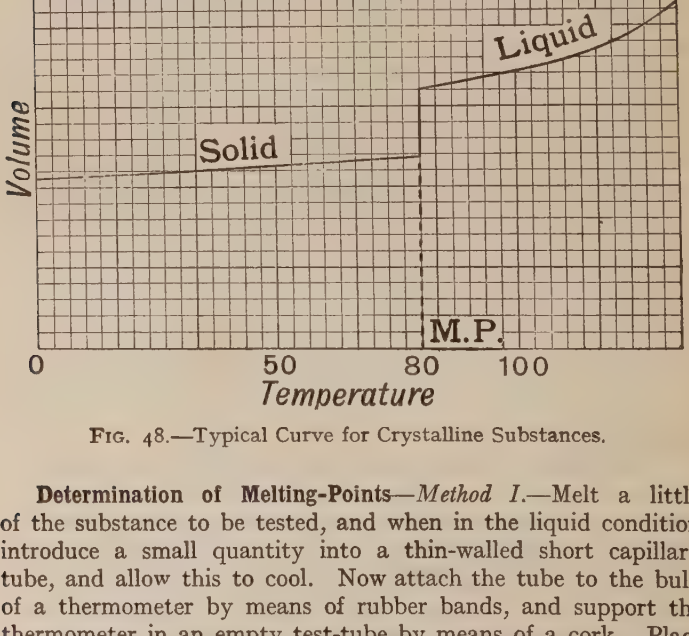

FIG. 48.—Typical Curve for Crystalline Substances.

Determination of Melting-Points—*Method I.*—Melt a little of the substance to be tested, and when in the liquid condition introduce a small quantity into a thin-walled short capillary tube, and allow this to cool. Now attach the tube to the bulb of a thermometer by means of rubber bands, and support the thermometer in an empty test-tube by means of a cork. Place this test-tube in a beaker of water, and slowly apply heat. The temperature at which the substance melts, as indicated by the transparent appearance it assumes, is carefully noted, and the flame is then withdrawn. Now let the substance cool, and note

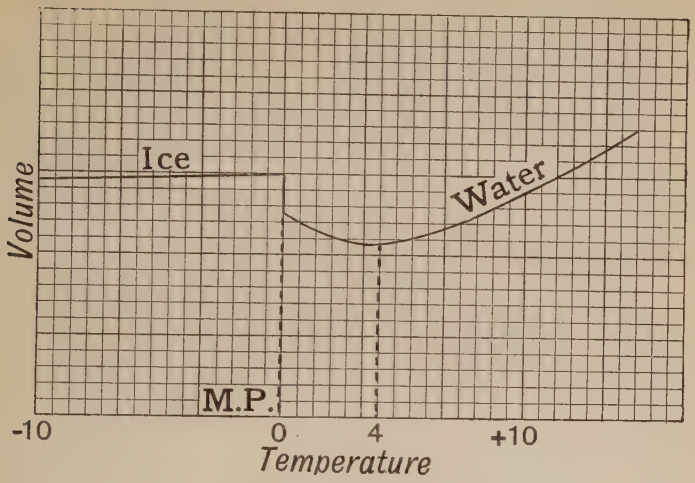

FIG. 49.—Curve for Ice.

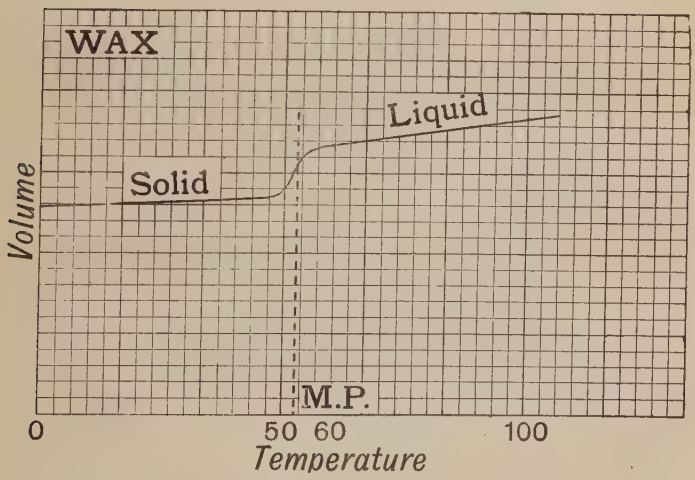

FIG. 50.—Typical Curve for Amorphous Substances.

the temperature at which solidification begins. The mean of the two readings so obtained gives the required results.

Method II.—The following is a simple method affording but a "rough and ready" determination of the melting-point. Melt a little of the substance in a shallow vessel, and immerse the bulb of a thermometer so that on withdrawal a thin film of the substance clings to it. Note the temperature at which this film solidifies. Now place the thermometer in a small beaker of water, apply heat, and note the temperature of melting. The mean of the two readings is then taken for the required result.

Method III.—By Supercooling.—This is a difficult but much more accurate method, the theory of which is very simple. Obtain some pure distilled water in a test-tube containing a thermometer and surround it with a freezing mixture of ice and salt. Keep the thermometer very still, and protect the apparatus from draught and all possible sources of disturbance. The water in the test-tube will become undercooled to perhaps $-5°$ C. Now suddenly stir the thermometer. Ice crystals will form, and at the same time the thermometer will rise to $0°$ C., thus indicating the freezing-point. The same method can be suitably modified for other liquids.

Method IV.—By a Temperature-Time Curve.—This method is often used for substances like paraffin-wax. Apparatus similar to that of Method I. above is suitable, the thermometer dipping directly into wax contained in the test-tube. The whole is heated to a temperature well above the melting-point, and the apparatus is then allowed to cool, temperature readings being taken every half-minute. A curve is now plotted from which the melting-point or region can be ascertained as indicated on page 47.

Melting-Point Values.—The melting-points of the commoner substances in general use are given in Table III. at the end of the book.

Effect of Pressure on Melting-Point.—The melting-points of substances are little affected by external conditions. Pressure changes have some appreciable effect, as will be obvious if it is remembered that the melting is usually accompanied by an increase in volume. The tendency of an increase of pressure is to resist the expansion, and therefore to hinder the melting, thus raising the temperature of the melting-point.

In the exceptional case of ice, and of those other substances which contract on melting, the application of pressure assists the melting process, and accordingly an increase in pressure decreases the melting-point. For ice the decrease is 0·0075° C. per atmosphere increase.

Regelation.—One consequence of the fact that the melting-point of ice decreases with increase of pressure is the phenomenon known as regelation (*i.e.* re-freezing). This is best illustrated by the following example:

AB (Fig. 51) is a block of ice over which are suspended two weights M (from 5 to 10 kilos) by means of a wire. The wire immediately begins to cut through the ice, and in the course of an hour or two will have gone completely through, *leaving, however, the whole block of ice intact as one piece.* The explanation is as follows:

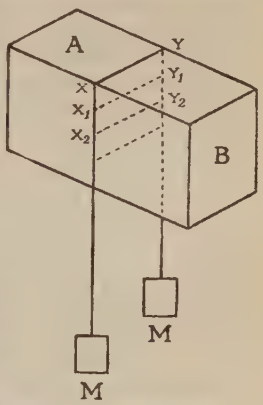

FIG. 51.—Regelation.

When the wire is first placed in position, the top layer of ice XY is subjected to a pressure due to the weights M. This lowers the melting-point below 0° C., and, as the temperature of the block is 0° C., the top layer at once melts, and the wire sinks through to the next layer. In doing so, however, the pressure is released from the layer XY, and consequently it once more freezes. The second layer is now subjected to a pressure, and it therefore melts. In this it is assisted by the latent heat which is given up by the first layer, and which is conducted down the wire. The wire thus sinks through to the third layer, which in its turn receives the pressure and the latent heat from the second layer, causing it to melt. The second layer being relieved of its pressure again freezes, and so on. In this way the wire is able to cut through the ice without altering the character of the block as a single piece.

The success of the experiment depends largely upon the material of the wire, for, as we shall see in Chapter XIII., some substances conduct heat much better than others. Copper wire

works very well indeed, iron cuts more slowly, whilst catgut is so bad a conductor that the heat does not travel downwards sufficiently quickly to assist the melting. The making of *snow-balls* is another example of regelation. On squeezing the snow, the pressure between the crystals comprising it causes melting, and the resulting water is forced out. This relieves the pressure, and the water thus formed refreezes on the outside, forming a hard binding mass.

The *motion of glaciers* is also explained by this phenomenon. The bed of a glacier is both sloping and uneven, and there are set up in the lower reaches and against jutting rocks pressures which are sufficient to produce melting. The next portion of the glacier slips down to take the place of the part thus melted, whilst the release in pressure from the melted portion causes it again to freeze. In this way the glacier slowly glides down at a rate of anything between 2 feet and 80 feet per day until it all melts.

Freezing Mixtures.—If we place some ice in salt water, the latter is cooled, *i.e.* it loses heat. This heat is absorbed by the ice, some of which therefore melts. By adding more ice, the reduction in temperature of the salt water may be continued until its freezing-point is reached. This will be below 0° C., the actual temperature depending upon the concentration of the salt, and the lowest possible temperature for the freezing-point obtainable with common salt is —22° C.

A mixture of ice and salt is therefore a useful means of producing cold, and is spoken of as a **freezing mixture.** The best proportions are three parts snow or ice-shavings to one part of common salt.

Another useful freezing mixture may be obtained by pouring powdered ammonium-nitrate into cold water (half a pound to a pint). In dissolving in the water it absorbs heat rapidly, and a temperature of −15° C. may be produced.

There are many other freezing mixtures. Three parts of calcium-chloride with two of snow can give a temperature of −55° C. Both liquefy, and in so doing they absorb much more heat than would be the case if only one were to liquefy; hence the very low temperature obtained.

EXAMPLES ON CHAPTER VI

1. What do you understand by "latent heat"? Explain the influence of latent heat in (*a*) supercooling, (*b*) regelation.
2. In what way do amorphous substances differ from crystalline substances with regard to melting and solidifying.
3. Draw curves showing the relationship between "change of state" and "volume." How is this relationship applied to the determination of the melting-points of substances?
4. Give an accurate method for finding the melting-point of napthalene.
5. How may the phenomenon of supercooling be utilised for the determination of the melting-point of a substance?
6. What influence has pressure on the melting-point of a sulphur? Describe experiments which have been performed to show this.
7. What do you understand by the term "regelation"? Explain the cause of the phenomenon, and state some of its applications.
8. How does the presence of a salt in solution affect the freezing-point of the liquid? How is this influence utilised?
9. What do you understand by "melting-point"? How would you find the melting-point of a butter-fat? If the melting-point of butter-fat is 31° C. to 36° C., and that of margarine is 40° C. to 43° C., what would you infer about a specimen of butter-fat which melts at 39° C.?
10. What do you understand by "fusion"? Mention a few of the commoner substances which "fuse" easily. How would you determine the melting-point of any one of them?
11. What do you understand by the term "temperature of change of state"? What inference would you draw from finding that the freezing-point of a given sample of liquid is a little lower than that given by a pure sample of the substance?
12. When a pavement or road is covered with ice during severe frosts, salt is sometimes thrown down in order to melt the ice. Explain why this happens, and support your explanation by describing a suitable experiment.
13. A piece of solid type metal floats on the surface of liquid type metal. Why is this, and what influence has this on the process of type-casting?

CHAPTER VII

CHANGE OF STATE—LIQUID TO VAPOUR

THE change of state from liquid to vapour may be brought about in two ways :

1. By the constant application of heat, *i.e.* by boiling.
2. By slow evaporation.

Boiling.—If we apply heat continuously to a liquid, we find that a steady rise in temperature is produced which continues up to a certain point. At this point the further application of heat produces no further rise in temperature, but instead the liquid begins to be converted into a vapour, *i.e.* it begins to boil ; all the heat energy is being utilised in the process of the change of state, and so long as there is liquid left and it is kept well stirred, the temperature remains constant. This constant temperature is spoken of as the **boiling-point.** The process of boiling is very interesting. As the temperature rises, bubbles are seen to form at the bottom of the liquid. These are probably of gases which have been dissolved in the liquid, and which become expelled as a result of the heating. They collect in the microscopic irregularities of the containing vessel, which act as nuclei for the formation of the bubbles. As these bubbles grow they become more buoyant, and soon they detach themselves and move upwards with the convection currents, diminishing in size as they rise, and perhaps collapsing with a slight clattering noise. Soon the temperature at the bottom of the liquid is at the boiling-point, and now the bubbles are almost entirely of vapour. Whenever there is irregularity on the inside surface of contact of the containing vessel, we get a point from which rises a constant stream of bubbles. This is due to the fact that a bubble requires a nucleus of some kind about which to form ; no bubbles start in the body of the liquid itself unless there is a speck of dust or

of some undissolved substance floating about in it; the effects of a few specks of sugar dropped into hot water are familiar to everybody.

This stream of bubbles is now big enough to penetrate to the surface and the clattering noise is softened into a "singing." The latent heat which before was imparted to the liquid by the bursting of the bubbles below the surface, and which therefore helped to raise the temperature, is no longer available for this purpose, since the heat is given off to the air, and the boiling of the liquid therefore takes place at constant temperature.

Overheating.—In process of time, irregularities on the glass surfaces of containing vessels become smoothed down, and the bubbles are unable to form for the want of nuclei. When this is so, it is possible to heat the liquid several degrees above the normal temperature of boiling-point without vaporisation taking place. In the case of water, the temperature will often go up to 105° C. or even 110° C., until eventually a row of bubbles suddenly form and burst with some violence, causing a "bumping," and the temperature again drops to 100° C. When there is a tendency to bumping, a little sand or any rough material placed in the vessel at once supplies the necessary nuclei for bubbles, and the boiling once more proceeds normally. An ideal smooth "containing vessel" may be obtained by surrounding a globule of the liquid by another liquid. *Dufour* introduced globules of water in a mixture of oils of suitable density and boiling-point, and was able to attain a temperature of 180° C. without the water boiling.

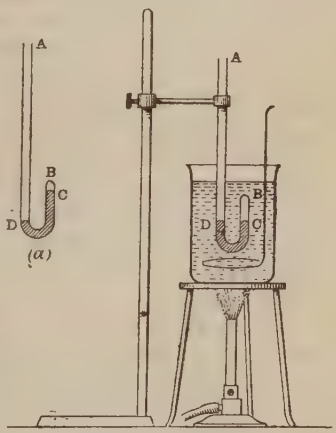

FIG. 52.—To show that Vapour Pressure at Boiling-Point is Atmospheric.

The Pressure of Vapour at the Boiling-Point.—What is the pressure exerted by a vapour given off by a liquid in boiling? A simple experiment will show this. AB (Fig. 52 (*a*)) is a bent tube

closed at B and open at A. Some alcohol (whose boiling-point is well below that of water) is introduced to fill the part BC, and is "trapped" by the mercury column CD, just reaching round the bend. The tube is immersed in a bath of water as shown in the figure, the end A being well above the surface, so that the pressure on the mercury at D is atmospheric. The water is now gradually heated, and the alcohol soon begins to evaporate. As a result the mercury level C is forced down and D is forced up.

Finally, when the boiling-point of the liquid is reached, it will be seen that the levels C and D are the same, **showing that the pressure exerted by the vapour coming from the boiling liquid is equal to that of the atmosphere.** This is true for all liquids. The experiment in the case of water is a little more complicated, the tube AB being surrounded with steam.

Vapour to Liquid—Condensation.—We have seen that during the process of boiling the temperature is constant. If instead of letting the steam escape we direct it by means of a copper delivery tube into a wider hard-glass or copper tube open to the atmosphere, then a bar-burner applied to the steam as it passes through this wider tube will raise its temperature considerably above 100° C. This can be followed by a thermometer inserted into it. The pressure of this *superheated steam,* as it is called, is still atmospheric. If we now take this superheated steam and cool it, *i.e.* extract heat from it, its temperature continually falls until a point is reached when the vapour begins to be converted into the liquid state. Throughout this process of liquefaction the temperature remains constant, and only when the process is completed will a further extraction of heat produce a further fall in temperature.

We speak of the vapour as being *condensed* into the liquid state, and the process is referred to as condensation.

A simple form of condenser is shown in Fig. 53. The water is boiled in the flask A, and the steam passes out through the delivery tube BC. This tube is surrounded by a cold-water jacket DD fitted with inlet and outlet tubes as shown, so that a steady circulation of cold water is maintained. The effect of this circulation is to extract heat from the steam as its passes along BC; the vapour condenses and it is collected in a vessel E. This is the method used for the purification or distillation of

water. The vessel A, sometimes spoken of as the still, contains ordinary water. This water holds in solution many salts and other impurities which in the process of boiling are left behind, and only the condensed steam is collected in E. The "hardness" of tap-water is due to the impurities it contains; pure water is

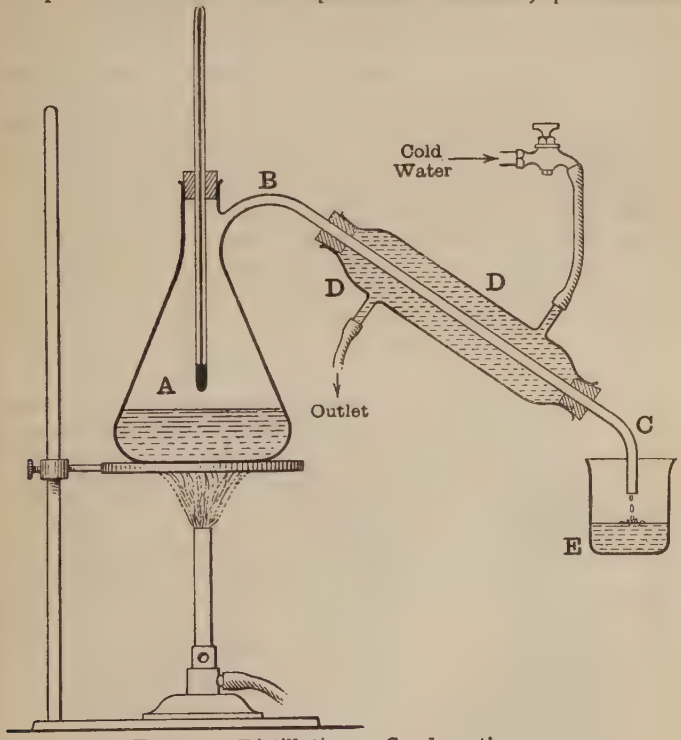

FIG. 53.—Distillation or Condensation.

very "soft." This process of distillation is used on steamships for the supply of fresh water to the boilers; the salt left behind in the still is, of course, periodically removed.

It frequently happens that the substance in the still is a mixture of different liquids having different boiling-points. Distillation affords a simple means of separating them, for the

constituent having the lowest boiling-point will be distilled off first, and then the boiling-point of another constituent will be reached, so that it too will now evaporate and be condensed and collected in its turn, and so on. Crude petroleum is such a mixture of substances, containing in increasing order of boiling-points the lighter oils (naphtha and illuminating oils), the heavier oils (used for lubrication), paraffin wax, and vaseline. Coal-tar is another interesting example. It is a black viscous fluid obtained as a by-product in the manufacture of coal-gas. Its constituents in ascending order of boiling-points are the coal-tar naphtha series (benzine 80° C., toluene 110° C., xylene 140° C., etc.), carbolic oil (phenol, naphthalene, etc.), creosote oil, anthracine oils, etc., leaving a residue of pitch in the still.

Various Boiling-Points.—A list of the boiling-points of some of the commoner liquids at atmospheric pressure is given in Table IV. at the end of the book.

Influence of Pressure on Boiling.—As in the case of freezing, the boiling-point is influenced by pressure changes. In consequence of the tremendous expansion of a substance on changing from liquid to gas (as in the case of water, 1 gram of which occupies about 1700 c.c. when converted into steam), the density of vapour is very small as compared with that of its liquid, and the influence of pressure on the boiling-point is far greater than on the melting-point. Since pressure opposes expansion, *the effect of increase in pressure is to delay boiling, and therefore to raise the boiling-point.* There is no exception to this.

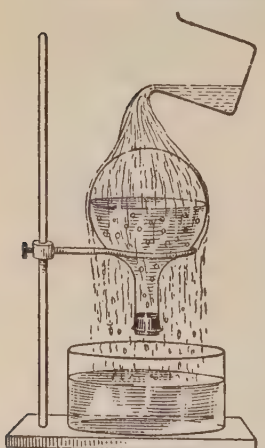

FIG. 54.—Water boiling under Diminished Pressure.

Robert Boyle investigated this effect as far back as 1665 by means of his air-pump. He introduced into the receiver some water whose temperature was somewhat lower than the normal boiling-point. On reducing the pressure in the receiver, he was able to reach a point at which the liquid started to

boil, so showing that reduction in pressure lowered the boiling-point.

The effect of pressure changes on boiling is clearly illustrated by the following well-known experiment. Boil some water for a time in a flask, and then tightly cork the opening. Invert the flask and support it in a ring-retort stand, as shown in Fig. 54. The space above the liquid is filled almost entirely with hot water vapour at atmospheric pressure. When the water has cooled a little below 100° C., so that boiling has ceased, pour some cold water over the flask. The sudden cooling is accompanied by a renewed boiling of the water at the diminished temperature. This is because some of the vapour is suddenly condensed into liquid, thus reducing the pressure of the vapour which remains.

Boiling-Points of Solutions.—If we introduce a salt solution into a flask provided with an outlet tube, and insert a thermometer so that its bulb is below the level of the liquid, we find that the boiling-point of the solution is greater than 100° C., *i.e. the effect of the solution is to raise the boiling-point.* By carrying out the experiment for different concentrations of salt, it will be found that the rise of the boiling-point is roughly proportional to the amount of salt dissolved.

The following table shows the concentration of common salt necessary per degree Centigrade rise in the boiling-point :

Boiling-Point of Water.	Percentage of Salt.
100° C.	0·0
101° C.	6·5
102° C.	12·0
103° C.	18·0
104° C.	21·5
105° C.	25·5

Note on Critical Temperature.—We have seen that as we increase the pressure to which a liquid is subjected, we delay the conversion of the liquid into the gaseous condition, *i.e.* we raise the boiling-point. This effect does not obtain indefinitely. As the pressure increases, a point is reached such that no further

additions to the pressure can have the slightest effect in further delaying vaporisation. *At the boiling-point corresponding to this temperature, therefore, we reach for any given liquid the maximum temperature of the liquid state, and above it none but a gaseous condition is possible.* This temperature is spoken of as the **Critical Temperature,** or the *Absolute Boiling-Point,* and the pressure corresponding to this temperature is called the *Critical Pressure.*

Slow Evaporation.—We began this chapter by pointing out that there are two ways in which a liquid can be converted into a vapour, namely, by boiling and by slow evaporation. We have dealt with the first of these at length, and we must now refer to the second; we shall consider it in detail in the next chapter. Slow evaporation is a phenomenon which differs from boiling in two important respects: firstly, it goes on at all temperatures, and, secondly, it goes on only at the surface of the liquid. Boiling, on the other hand, occurs only at a specified temperature, and goes on throughout the body of the liquid. The properties of the vapour produced are, however, the same in both cases.

If we leave a little water or any other liquid in a small shallow vessel we find that it gradually disappears. Those of the surface molecules which are at any instant moving in an upward direction with a sufficient velocity will leave the liquid, and as this is constantly occurring, the dish in time becomes quite dry. We speak of this as evaporation.

Loss of Heat on Evaporation.—As in the case of the conversion of solids to liquids, no liquid can become vaporised without heat being supplied to it. This heat does not increase the temperature of the substance, since it is all being utilised in the process of the change of state. We speak of this heat as **the latent heat of vaporisation,** to distinguish it from the *latent heat of fusion* which is necessary for the conversion of a solid to a liquid. The former quantity is considerably greater than the latter; in the case of water, for example, the latent heat of vaporisation is nearly seven times as great as the latent heat of fusion (see Chapter XI.).

When, therefore, it is desired to convert a liquid into a vapour, if we do not supply the heat artificially, the change of state is

only rendered possible by the absorption of heat from the surroundings, which are therefore cooled. There are many examples of this cooling produced by evaporation. The following well-known experiment illustrates it very well. Take a flat piece of wood and pour some water on to it to form a little pool. On this place a beaker containing some ether. Dip a glass tube into the ether, and bubble air rapidly through the liquid. Ether is very volatile, and rapid evaporation takes place. The heat which must be absorbed for this evaporation comes from the surroundings, *i.e.* it is extracted from the film of water between the wood and the beaker. The effect of this extraction of heat from the water is to freeze it, and it soon becomes possible to lift the beaker, ice, and wood as one mass.

Sublimation.—There is yet a third change of state to which a reference should be made—that from a solid to a vapour direct, without the intermediate liquid state, thus it is well known that camphor slowly evaporates away at the temperature of the atmosphere. We are conscious of this evaporation in the *smells* of the bodies in question. The converse is also true—the vapours on cooling pass into the solid stage without first liquefying. This process is spoken of as sublimation; "flowers of sulphur" is produced in this way from sulphur vapour.

EXAMPLES ON CHAPTER VII

1. Trace in detail the behaviour of ice which from a temperature of $-5°$ C. is steadily heated to a temperature of 110° C.
2. Describe carefully the process of boiling. In what way does the nature of the containing vessel contribute to this phenomenon?
3. What is the cause of the "bumping" which sometimes occurs in boiling? How can it be remedied?
4. What do you understand by superheated steam? How can it be produced at atmospheric pressure?
5. How could you show experimentally that the pressure exerted by the vapour coming from a boiling liquid is equal to that of the atmosphere?
6. How may absolutely pure water be obtained from an impure sample?
7. Give an account of some of the industrial applications of the principles of distillation.

8. Give details of the method you would employ for the determination of the boiling-point of an inflammable liquid such as benzine.
9. Distinguish ebullition from evaporation. Why is it that at high altitudes water boils at a lower temperature than at the sea-level ?
10. Why is it necessary when fixing the boiling-point of a mercury thermometer to know the height of the barometer ? Explain how, with the aid of a thermometer, it would be possible to estimate the height of a mountain.
11. What is the effect of pressure on the boiling-points and melting-points of substances ? Describe experiments to illustrate your answer.
12. Give an accurate method for determining experimentally the influence of pressure changes on the boiling-point of a liquid.
13. How can the influence of pressure on the boiling-points of liquids be applied to the maintaining of a constant temperature ?
14. How would you determine experimentally the influence of the concentration of a salt in solution on the boiling-point ? Would you expect sea water and river water to boil at the same temperature ?
15. What do you understand by the term "latent heat of vaporisation" ? Describe and explain some simple experiment to illustrate the cooling effect produced by evaporation.
16. What change of state do you know of in addition to melting and boiling ? What evidences are there of the existence of this third change of state ?
17. What is meant by the latent heat of vaporisation of water ? "A Scotch shepherd, if his plaid is partly wet through, and he is out at night, turns his plaid with the wet side in." Explain why this proceeding is a safeguard against chills, and describe any one experiment to illustrate your answer.
18. In tropical climates drinking water is stored in a suspended unglazed earthenware jar, and is allowed to filter through and to drip into a lower and smaller glazed jar. Why is this ?
19. A thermometer is dipped into a vessel containing alcohol, and reads 52° F. It is then withdrawn and held in the air. The temperature reading at once falls a little. Why is this ?
20. Steam at 212° F. produces more severe burns than water at the same temperature. Why is this ?
21. Discuss the influence of body perspiration on the temperature of the human body.

CHAPTER VIII

PROPERTIES OF VAPOURS

Saturated and Unsaturated Vapours.—If we take a simple barometer tube AB (Fig. 55), and fill it with mercury, and in the usual way invert it and place the end B under the mercury level of the reservoir C, we get a column DB resulting, which balances the air pressure. Above D there is practically a vacuum. Suppose we now take a pipette E with a bent extremity, and with it we introduce a very small quantity of some liquid. Being much lighter than the mercury, it is at once forced to the top as a thin film over D, and here *it immediately evaporates*.

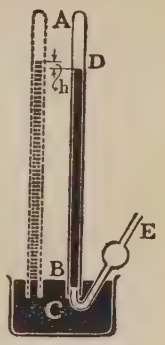

FIG. 55.

Why is this? The explanation is very simple, and is furnished by the Kinetic Theory. The molecules of the liquid are in haphazard motion, and those directed upwards encounter no resistance. They accordingly leave the liquid and move about in the space above, presently to be joined by other molecules, until in a very short time no liquid is left; it has all been converted into vapour, and the molecules behave exactly as do those of a gas.

Now suppose we introduce a further small quantity of the liquid above the mercury column by means of the bent pipette. The same thing happens; very quickly the whole of the liquid becomes vaporised, the molecules occupying the space AD. We have direct evidence that this is so, for if we place another barometer tube side by side with it, and compare the barometric heights in the two cases, we find that the level of D has been depressed by a small amount h due to the pressure of the vapour produced by the "molecular bombardment" on D.

If we go on introducing very small quantities of the liquid by means of the pipette, we soon find that a limit is reached such that no more liquid becomes vaporised; instead it remains as a thin film over D. When this stage is reached, *we say that the vapour above it is* **saturated**; before this limit is reached the vapour is spoken of as being *unsaturated*.

Explanation of Saturation by the Kinetic Theory. — To understand the cause of saturation, it must be remembered that haphazard motion obtains both in liquids and in gases. When a film of liquid is introduced above D, not only do some of its molecules tend to move up out of the liquid into the vapour already above it, but there are also molecules of the vapour above moving directly downwards into the film of liquid itself. Hence something in the nature of an exchange of molecules is going on. In the early stages, however, when very little liquid has been introduced above D, the number of molecules moving up from the liquid is much greater than the number of vapour molecules moving down into it, so that the result as a whole is evaporation. As more and more vapour is produced, however, the tendency for molecules to move downwards increases, and eventually a stage is reached when there are as many molecules moving down into the liquid as there are moving up from it, the net result being that no more vapour is produced: the amount above D has reached its maximum value.

Vapour Pressure.—We have seen that the presence of the vapour is demonstrated by the pressure it exerts upon the surface D of the mercury (Fig. 55). As more and more unsaturated vapour is produced, its density increases, and hence its pressure also increases, until when the stage of saturation is arrived at, the vapour density, and hence the vapour pressure, attains its maximum value. This is spoken of as the **saturation pressure,** or the *saturated vapour pressure*. It is measured by the difference in levels *h* in Fig. 55.

Influence of Air on Evaporation.—We have so far considered the phenomena of saturated and unsaturated vapours with regard to confined spaces, as in the space AD. But such limitations are quite unnecessary. We saw in Chapter VII. that if a liquid is left in a shallow dish exposed to the air, it slowly evaporates until the dish is quite dry. The process by

which the liquid molecules are converted into an unsaturated vapour is identical with that just described as taking place in the vacuum AD.

It may be thought that the pressure of the atmosphere should modify the process. It does so, however, only in the sense that the rate of evaporation is much slower in air than in a vacuum. Ultimately exactly the same amount of liquid will evaporate into a space filled with air as would evaporate in the same space as a vacuum. The retarding influence of the atmosphere on evaporation is very beneficial to us. It should be remembered that by far the larger portion of the earth's surface is water. Evaporation is continually going on, and if it were not for the atmosphere a perpetual condition of saturation would result, and such things as dry clothing would be impossible under normal conditions. Owing to the air, however, a state of saturation exists only under certain occasional conditions which we shall study later (Chapter IX.). Normally the atmosphere contains only unsaturated vapour.

Properties of Unsaturated Vapours.—Vapours are by their nature so very much akin to gases, that we are obviously led to an investigation as to how far they obey the Gas Laws. We will first deal with unsaturated vapours.

To test Boyle's Law for unsaturated vapours, the apparatus described in Fig. 55 is sufficient if we substitute for the reservoir C one much deeper. Beginning with the mercury levels equal, we introduce a very small quantity of liquid above D, so that it all immediately evaporates. The levels are no longer equal, the difference h giving us the pressure exerted by the unsaturated vapour filling the space AD. There is also of course a very little mercury vapour over both columns, but at the temperature at which the observations are taken its effect may safely be neglected. Now push the tubes lower down into the reservoir. Since the column BD plus the vapour pressure has still to balance the atmospheric pressure, the tendency is for the mercury to rise and to compress the vapour into a smaller volume. This will be found to affect the value of h. If the pressures and volumes of the vapour are thus noted under varying conditions, it will be found that their product is constant. **Boyle's Law is therefore obeyed by unsaturated vapours.**

We have now to test the vapour for Charles' Law. To do this we surround the vapour with a water-bath as shown in Fig. 56.

Fig. 56.—Vapour Pressures at Varying Temperatures.

The front of the bath is glazed so that the levels of the mercury in the two tubes may be accurately read by a cathetometer. A spirit-lamp or Bunsen burner raises the temperature of the water to any desired amount, whilst a stirrer is employed to keep it uniform throughout. The experiment consists in noting the variations of volume with temperature, and as a result we find that **Charles' Law holds for unsaturated vapours.**

We see therefore that in nearly every respect unsaturated vapours behave as gases, and for purposes of calculation the Gas Laws may always be applied to them.

Relation between Pressure and Volume for Saturated Vapours.—If we repeat the test or Boyle's Law described above, using a saturated instead of an unsaturated vapour, we find that as the tube AB (Fig. 55) is lifted up and down in the reservoir in different positions, thereby subjecting the vapour to different volumes, the pressure remains unaltered.

The pressure of a saturated vapour is therefore independent of its volume.

This is fully in accord with the Kinetic Theory, for it will be remembered that the presence of saturated vapour

implies a condition of kinetic equilibrium between the molecules of the liquid and those of the vapour; as many molecules are leaving the liquid as are entering it from the vapour. If we diminish the volume of the vapour, we increase the bombardment of the surface of the liquid, so that more molecules leave the vapour to form the liquid than leave the liquid to form the vapour. Hence instead of the diminution in volume producing an increase in pressure, as would be the case if Boyle's Law were obeyed, it produces a condensation of some of the vapour until the pressure is reduced to the normal amount, and kinetic equilibrium between the molecules is once more established. The converse happens if we increase the volume. In this case more molecules leave the liquid than enter it, so that evaporation is produced until the pressure attains the normal value, and kinetic equilibrium between liquid and vapour is again restored.

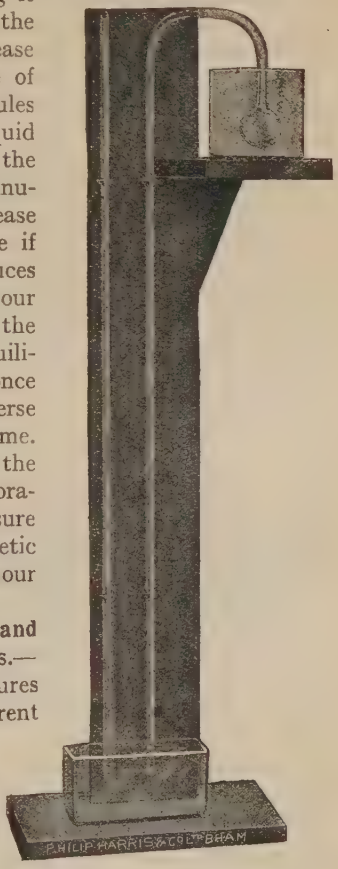

FIG. 57.—Vapour Pressures below Zero.

Relation between Pressure and Temperature for Saturated Vapours.—To determine the vapour pressures of saturated vapours at different temperatures, different forms of apparatus are necessary according to the range of temperatures involved. For ordinary temperatures between 0° C. and 50° C., the apparatus shown in Fig. 56 answers well. For temperatures below 0° C., Gay-Lussac used two barometer tubes both dipping into the same reservoir, but the upper extremity of that containing the vapour

was bent and terminated in a bulb which was immersed in a freezing mixture (Fig. 57).

If we plot a graph connecting the maximum vapour pressures with temperatures, we get a curve of the type shown in Fig 58—concave upwards. The importance of the shape of this curve will be shown in Chapter IX., when we deal with hygrometry. The figure shows curves for both alcohol and water. It will be noticed that even at 0° C. there is quite an appreciable vapour

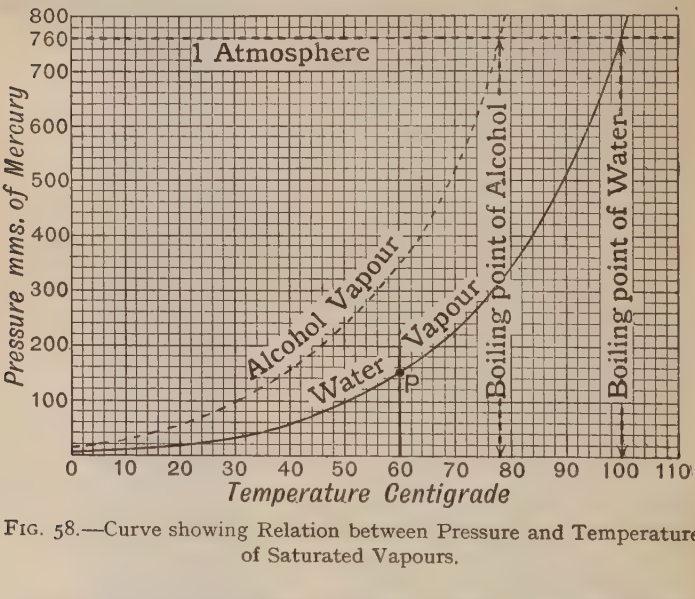

FIG. 58.—Curve showing Relation between Pressure and Temperature of Saturated Vapours.

pressure, and that the pressure rapidly increases with temperature until at the temperature of the boiling-point the pressure is atmospheric. This it will be remembered was experimentally proved on page 89. It follows as an immediate consequence of this that if we have two liquids A and B such that A boils at a higher temperature than B, then at any given temperature the saturated vapour pressure of A will be less than that of B. Mercury, which boils at 357° C., accordingly has a very small vapour pressure at normal temperatures.

Numerical Examples on Vapour Pressures.—Mixtures of gases or of vapours or of gas and vapour are easily dealt with, since we may assume the combined pressure to be equal to the sum of the pressures of the constituents.

Example.—*When* 50 *c.c. of a gas are collected in a tube over water at a temperature of* 12° *C., the height of the water column below the gas is* 20 *cms. above the level of the water in the trough. What is the volume of the dry gas at N.T.P. if the barometer reads* 740 *mm. ?*

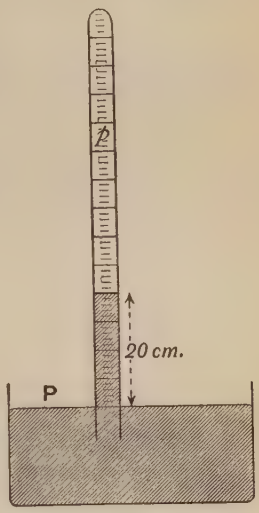

FIG. 59.

Solution.—Let us denote the pressure of the gas by p, and atmospheric pressure by P (Fig. 59).

Then $P - p = 20$ cms. of water

$$= \frac{20}{13{\cdot}6} \text{ cms. of mercury.}$$

$$\therefore \quad p = 740 - \frac{20 \times 12}{13{\cdot}6}$$

$$= 725{\cdot}3 \text{ mm. of mercury.}$$

Now this is made up of the pressure of the dry gas plus the saturation pressure of moist water vapour at 12° C. From the table of saturation pressures in the Appendix, we see that the saturation pressure of water vapour at 12° C. is 10·43 mm.

$\therefore$ Pressure of dry gas at 12° C. $= 725{\cdot}3 - 10{\cdot}43$
$= 714{\cdot}87$ mm. of mercury.

We now require the volume of this dry air at N.T.P.

We have

Old pressure $= 714{\cdot}87$ mm.
Old volume $= 50{\cdot}0$ c.c.
Old temp. $= 12 + 273°$ A.

and

New pressure $= 760$ mm.
New volume $= V_1$.
New temp. $= 0 + 273°$ A.

Since $$\frac{PV}{T} = \frac{P_1V_1}{T_1}$$

we have $$\frac{714{\cdot}87 \times 50}{285} = \frac{760 \times V_1}{273}$$

Whence $$V_1 = \frac{273 \times 714{\cdot}87 \times 50}{285 \times 760}$$

$$= 45{\cdot}03 \text{ c.c.}$$

Vapour Pressure when the Containing Vessel is not Uniformly Heated.—If we have a vessel containing liquid and vapour (Fig. 60) such that a portion A is at a temperature of $t°$ C. higher than that of another portion B at $t_1°$ C., initially the vapour pressures will be different at the two places, for we have seen that the pressure increases with the temperature.

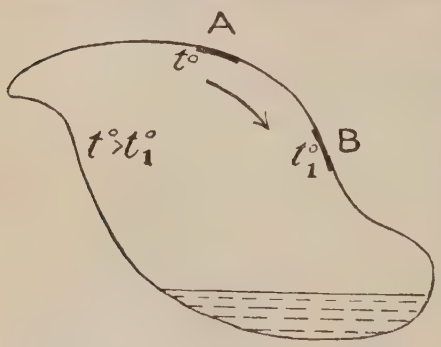

Fig. 60.

The excess of pressure at A over that at B will induce a current of vapour from A to B. The molecules at A at a saturation pressure p_t will on moving to B come to a region where the saturation pressure p_{t1} is lower than p_t, and condensation will therefore take place, the condensed particles running down to the liquid below. This will continue until the whole of the pressure has been reduced uniformly to the lower value p_{t1}. Hence it follows that *if we have a saturated vapour in a vessel whose temperature differs in different parts, provided the vapour can move freely from one part to another, the pressure on the vessel is that which belongs to the lowest temperature.*

Wollaston's Cryophorus (Fig. 61) is an application of this. Two bulbs A and B are connected by a horizontal tube. The bulb A contains water, and all the air has been removed from the

remainder of the space. Bulb B is immersed in a freezing mixture C, causing a difference of temperature in the parts of the apparatus, and hence an excess of vapour pressure in A over that in B. A current of vapour therefore passes from A to B, and the temperature in B is sufficiently low for condensation to take place steadily, thus

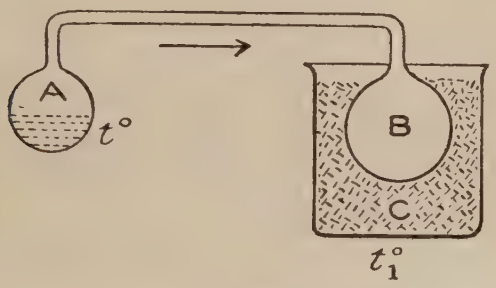

FIG. 61.—Wollaston's Cryophorus.

maintaining the difference of pressures, and therefore the current of vapour.

This process does not go on indefinitely, however, for it must not be forgotten that in condensing in B an absorption of heat must occur, and this heat is extracted from the bulb A, which therefore persistently cools until the water left in it freezes.

We shall see in the next chapter how this principle is applied to meteorological instruments.

EXAMPLES ON CHAPTER VIII

1. Define the terms "unsaturated vapour," "saturated vapour," "vapour pressure." How would you show the effect on its pressure of a diminution in the volume of (*a*) an unsaturated vapour, (*b*) a saturated vapour ?
2. How can it be shown that the pressure of a saturated vapour is unaffected by the presence of a gas such as air ?
3. Describe an experiment for the determination of the vapour density of an unsaturated vapour, and explain the theory of the method you describe.

4. A barometer tube dipping into a mercury reservoir contains a mixture of air and saturated vapour above a column of mercury which is 70 cms. above the tube in the reservoir, the atmospheric pressure being 76 cms. What is the height of the mercury column when the air is depressed so as to reduce the volume occupied by the air to half its original value, the pressure of the saturated vapour being 1·5 cms. ?
5. Why does a liquid vaporise much more slowly when only one degree below the normal boiling-point ?
6. What atmospheric conditions will (*a*) favour evaporation, (*b*) cause a change in the boiling-point of the liquid ? Give reasons for your answer.
7. How would you proceed to investigate the validity of Boyle's and Charles' Laws for an unsaturated vapour ?
8. What is the relation between the boiling-point and the vapour pressure of a liquid ?
9. How does the presence of water vapour affect the pressure of a gas ? Indicate this graphically by drawing an isothermal (*a*) for a dry gas at $t°$ C., and (*b*) for damp air at $t°$ C.
10. Enumerate the points of resemblance and difference between a gas and a vapour.
11. A and B are two barometers. A has a little air above the mercury, while B has a little air and a drop of water. The readings of the barometers happen to be equal at the temperature of the room. Will they be equal when the temperature is raised or lowered, and if not, which will give the higher readings ?
12. Describe and explain the action of the cryophorus.
13. Describe carefully the difference between evaporation and boiling. What effect has the presence of the air above in each case ? Why does ether boil at a lower temperature than water ?
14. 100 c.c. of gas are collected over water at 15° C., leaving a column of 23 cm. of water in the tube above the level of the water in the trough. What is the volume of the dry gas at N.T.P. if the barometer reads 745 mm. ?
15. 82·4 cubic inches of gas are collected at 52° F. when the barometer stands at 29·75 inches. Find the volume of the dry gas at N.T.P.
16. Gas is collected in a vessel over a liquid whose vapour pressure is 12·5 mm. The volume registers 735 c.c. and the temperature is 16° C. If the barometer reads 765 mm., what is the volume of the dry gas at N.T.P. ?
17. How would you demonstrate that sulphuric acid has practically no vapour pressure at ordinary temperatures ?

18. Describe how you would measure the vapour pressure of ether at 68° F.

19. The following table shows the maximum vapour pressures of water vapour in mm. of mercury :

Temperature.	0° C.	10° C.	20° C.	30 C.°	40° C.	50° C.
Vapour Pressure (mm. of mercury)	4·6	9·2	17·4	31·6	54·9	92

Temperature.	60° C.	70° C.	80° C.	90° C.	100° C.
Vapour Pressure (mm. of mercury)	148·8	233·1	354·6	525·4	760

Illustrate by means of a graph, and discuss the result obtained. From it find (*a*) the vapour pressure at 37° C., and (*b*) at what temperature the maximum vapour pressure is 200 mm.

CHAPTER IX

HYGROMETRY

Water Vapour in the Atmosphere.—We now proceed to the study of the presence of the water vapour in the atmosphere. A condition of the air involving absolute absence of moisture is impossible, owing to the fact that five-sixths of the earth's surface is water. Whatever may be the atmospheric conditions of temperature and pressure, molecules must constantly be leaving the water surface to intermingle with the air molecules above. We have seen that if it were not for the presence of the air, there would be a perpetual condition of saturation. Owing, however, to the retarding influence of the atmosphere in delaying the rate of evaporation, it is only under occasional physical conditions that a state of saturation will exist. The study of the influence of water vapour on the atmosphere is known as *hygrometry*.

Temperature and Saturation.—It will be remembered that when we have a surface of separation of liquid from vapour, there is a constant interchange of water- and vapour-molecules. Some of those from the vapour continually become entangled amongst those of the liquid, and *vice versa*. When this exchange goes on at equal rates, the net result is that the total quantity of vapour remains unaltered, and we say that it is in a saturated condition. If, however, the rate at which the molecules leave the vapour for the liquid exceeds that at which molecules leave the liquid for the vapour, the quantity of the latter as a whole diminishes, and we say that *condensation is taking place*; and, finally, if fewer vapour molecules enter the liquid than leave it, the total quantity of vapour must be increasing, and we say that evaporation is taking place.

The chief factor which in the atmosphere makes for the occurrence of condensation or evaporation is temperature. Increase of temperature increases the molecular motion of the substance

heated. For gases and vapours, the kinetic energy of the molecules is roughly proportional to the absolute temperature, and it is extremely probable that the increase in the kinetic energy of the liquid molecules goes on at a much greater rate than this. When, therefore, we raise the temperature slightly, evaporation "has the upper hand" over condensation until sufficient liquid has evaporated to produce equilibrium. Conversely, on cooling, condensation will take place.

Dew.—Suppose we have a mass of air containing a certain amount of water vapour in an unsaturated condition at $t°$ C. This may be represented in a pressure-temperature diagram (Fig. 62) by a point P *below* the saturation pressure line, *i.e.* the vapour pressure at this temperature is not high enough for the air which contains it to be saturated. If, however, we now cool the air, since the amount of water vapour present remains unaltered, its pressure is constant. Hence, as the air cools we move from the point P along the constant pressure line PQ, arriving finally at a temperature $t_1°$ C. such that the point Q is *on* the saturation line; *i.e.* at this lower temperature the pressure is now sufficient to produce saturation. Further cooling brings us to the *left* of the saturation line, which is in the region of the liquid state, and condensation takes place.

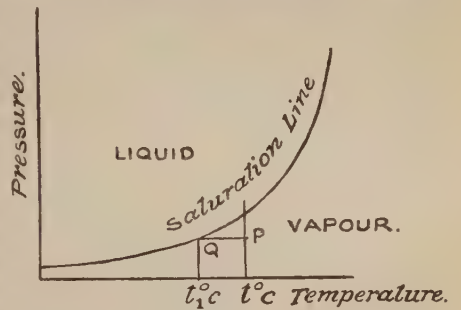

FIG. 62.

Owing to the rotation of the earth on its axis, the atmosphere experiences a diurnal cycle of temperature changes, and while during the daytime the temperature may be sufficiently high for the water vapour present in the air to be below the saturation pressure, as the evening approaches the temperature falls, and such objects lying on the ground as are good radiators, *i.e.* bodies which readily emit their heat to their surroundings (see Chapter XIV.) cool more quickly than other bodies which are bad radiators.

Consequently the air in contact with these cools too, and if this cooling is sufficiently great, the saturation pressure will be reached, and a film of moisture will be condensed on the surface of the bodies. We speak of this deposit of moisture as **dew, and the temperature to which a given sample of air must be cooled for dew to be deposited is called the dew-point.**

The best substances for the deposition of dew are those which are both good radiators and bad conductors (see Chapters XIII. and XIV.), for while the earth is giving up its heat, as in the evenings, if a good radiator is also a good conductor, then as fast as it is radiating its heat away to the space above it is taking more in from the cooling earth. Wood is on this account a much better surface for the deposit of dew than metal.

The formation of dew is greatly hindered if the air is in rapid motion, for in that case no portion of the air remains over a cooling object sufficiently long for it to be cooled below the dew-point. On the other hand, very still air also hinders the formation of dew, for air is such a bad conductor that it cools very slowly indeed when it is quite still. Again, the presence of clouds interferes with the deposition of dew, for they act as a screen to prevent the heat from being radiated out into space, and thus hinder the necessary fall in temperature. A fairly calm, clear sky therefore offers the best conditions for dew. Finally, it should be noted that an object fairly high above the earth does not as a rule receive a deposit of dew, for as soon as the air in contact with it cools, but before it has time to cool below the dew-point, its increased density causes it to sink towards the earth, other air taking its place.

Condensation above the Earth's Surface—Fog and Mist.—It has been definitely established that condensation of vapour most easily occurs when there are some nuclei about which liquid drops can form. Such nuclei are provided in the atmosphere by the very large number of dust particles which are everywhere held in suspension, no matter how clear the air may be. Much valuable work on this subject has been done by *Aitken*, who perfected an apparatus for counting the number of dust particles in any given sample of air. Aitken showed that in the country districts the number is far fewer than in the town areas, but that even the purest of atmospheres is not entirely free from them.

A sample of air which is almost free from dust particles could

be cooled down many degrees below the normal dew-point without condensation taking place, and the condition is then one of *super-saturation.*

If for any reason the air some little distance above the ground becomes cooled below the dew-point, the condensation becomes visible as *mist.* Over manufacturing towns the soot particles which are constantly being poured into the atmosphere from chimneys give rise to the particular variety of mist known as *fog.* Fogs occur chiefly in the winter months, when the night atmosphere loses a great amount of heat, and large masses of air become cooled below the dew-point. The only difference between a fog and a mist is that the one is much thinner than the other. This is because the nuclei in the country mists are generally so much smaller than the dust and soot particles over the towns.

Mixtures of Air at Different Temperatures.—One of the chief causes which give rise to atmospheric cooling is the mixing together

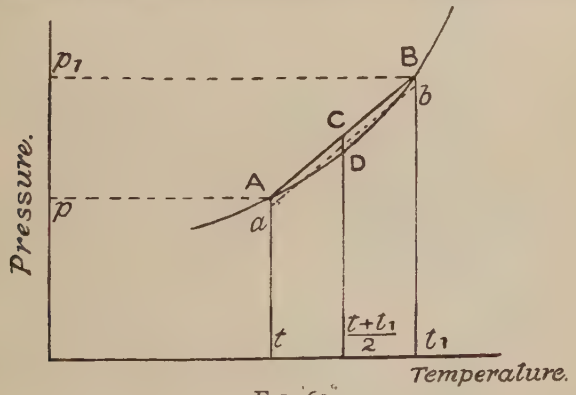

FIG. 63.

of different masses of air at different temperatures. This is a consequence of the fact that the shape of the pressure-temperature curve for water-vapour is concave upwards. Suppose we have two equal masses of air at temperatures t and t_1 respectively, and suppose that each mass contains vapour at its saturation pressure, so that their positions on the graph are represented by A and B respectively (Fig. 63).

If these masses are now brought together, assuming t_1 is greater than t, the one is heated and the other is cooled, and we shall see in the next chapter that the resulting temperature is the mean of the two, *i.e.* $\frac{t+t_1}{2}$. The resulting pressure will also be the corresponding mean $\frac{p+p_1}{2}$, as indicated by the point C in the figure. This, however, is in excess of the saturation pressure at the temperature $\frac{t+t_1}{2}$ by the length CD on the pressure scale, and brings the substance into the "liquid area." It is obvious that condensation must result until the pressure is reduced to that of the point D.

It will be noticed that even if the original pressures of the two portions of air were not at the saturation values, but just a little below, as with the points *a* and *b* in the diagram, we should still get condensation as a result of the mixture of the two portions, so long as the line *ab* (shown dotted in the figure) cuts the mean ordinate between C and D. Hence, **when two portions of air at different temperatures and at or near their respective saturation pressures are brought together, condensation will occur.**

Clouds, Rain, Snow, etc.—*Clouds* are often formed in this way from such air-movements as winds, and if the condensed drops are sufficiently large for the force of gravity to overcome the viscosity of the air, they fall to the ground as *rain*. It happens sometimes that the temperature at which condensation takes place is below zero, and when this is so, instead of rain clouds we get snow clouds, with falls of *snowflakes* instead of rain. Instead, too, of a deposition of dew on the ground, we get *hoar frost*. *Hail* is produced by the freezing of falling raindrops as they pass through a stratum of air below 0° C.

Measurement of Dew-Point.—Various forms of apparatus are in use for the determination of the dew-point. These all act on the same principle. The air surrounding some bright surface is cooled until moisture begins to be deposited. This shows itself as a dimming of the surface. The temperature at which this dimming takes place is noted, but as this is probably too low a reading, the source of cooling is now removed, and the tempera-

ture at which the moisture is seen to disappear is read. The mean of these two readings gives the desired dew-point. The different forms of apparatus differ only in the mechanism adopted for producing the cooling.

Daniell's Hygrometer.—One of the oldest forms is that known as Daniell's Hygrometer. This is really a cryophorus, consisting of two bulbs connected by a tube bent as shown (Fig. 64), and supported on a stand carrying a thermometer. Some ether is introduced into the lower bulb, and its temperature is recorded by a thermometer supported in the left-hand limb of the instrument. All the air having been withdrawn, the tube is sealed up, and it thus contains nothing but ether and ether-vapour. A muslin rag (not shown in the figure) is now tied round the right-hand bulb, and a little ether is poured on to it. Its evaporation produces cooling due to the absorption of latent heat. As with the cryophorus, differences of vapour pressure are set up, causing a flow of vapour to the right-hand bulb, and a lowering of the temperature of the left-hand bulb. By pouring more ether over the muslin rag, this cooling is continued until a gilded band on the left-hand bulb shows a film of moisture. Readings are taken, and the process is reversed, as already explained. Most Daniell hygrometers work badly, and greatly try the patience of the experimenter, but if the instrument is small, and does not contain too much ether, it should give consistent results, the two readings with practice being not more than half a degree apart. The instrument must be exposed to neither draught nor sunshine, nor to the experimenter's breath or perspiring fingers. This applies to all hygrometers.

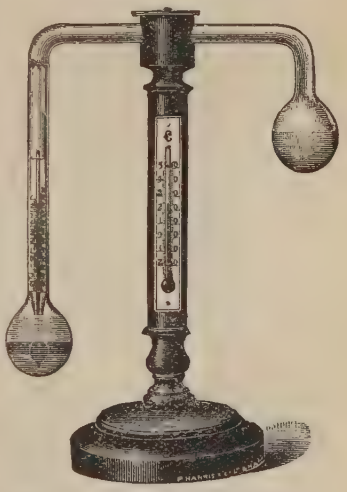

Fig. 64.—Daniell's Hygrometer.

Regnault's Hygrometer.—This consists of a test-tube to the lower portion of which is fitted a silver cap containing ether. The temperature of the ether is read by a thermometer immersed in it. (Fig. 65.)

The ether is connected with the atmosphere by a thin tube

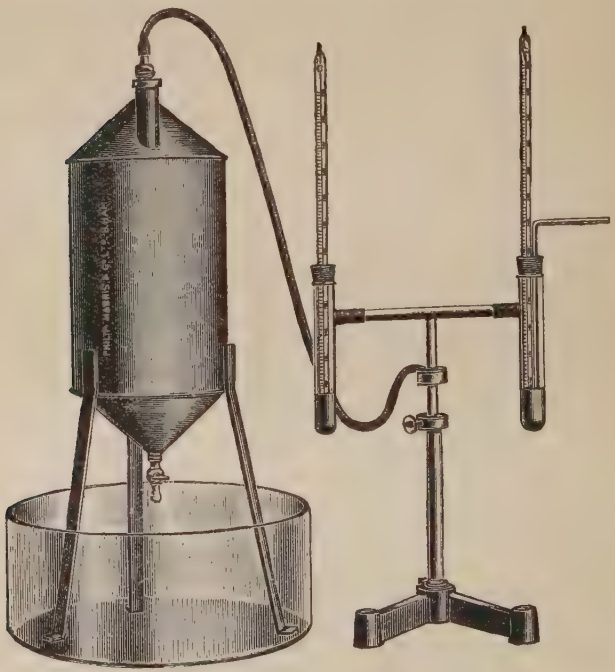

FIG. 65.—Regnault's Hygrometer.

dipping into it from outside the test-tube, and with an aspirator by a side-tube leading from the test-tube. By turning on the tap of the aspirator, air is bubbled through the ether, causing evaporation and cooling, and, as before, readings are taken as soon as condensation is detected. To facilitate this a second tube and silver cap is placed alongside the first for purposes of comparison. One of the chief advantages of this instrument is

the ease with which the rate of cooling is regulated by the aspirator. In another form of this apparatus a box is glued to the back of a thin, dark glass-plate or piece of polished metal. The box contains ether, and air is bubbled through it.

Wet and Dry Bulb Hygrometer.—This consists of two simple mercury thermometers (Fig. 66), the bulb of which is wrapped in one end of a linen rag. The other end of the rag is immersed in a well of water. By capillary attraction this linen "wick" keeps the bulb of the right-hand thermometer constantly wet, and therefore evaporation will be going on, accompanied by the absorption of heat. This will be indicated by the lower thermometer reading. The drier the air, the greater will be the rate of evaporation of the moisture, and therefore the greater will be the difference between the two thermometer readings. When, however, the air is already near the condition of saturation, the rate of evaporation from the right-hand bulb will be slow, and the thermometers will read very nearly alike. It should be remarked that the readings are more reliable if the air is made to circulate round the two bulbs. There exists a relationship between the difference in the temperature readings and the dew-point, which is expressed by an empirical formula, *i.e.* a formula not proved mathematically, but found as a result of experiment. From this formula tables are drawn up, and by a reference to these the dew-point corresponding to any given set of observations can be found.

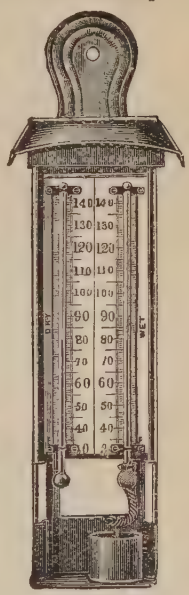

FIG. 66. — Wet and Dry Bulb Hygrometer.

Example 1.—*On a certain day a wet and dry bulb hygrometer shows a reading of* 17° *C. for the dry bulb, and* 14° *C. for the wet bulb. Find the dew-point.*

Solution.—Difference in readings = 3° C.

Referring to Table V. at the end of the book, we look down the first column, which refers to the dry bulb readings, and confine

our attention to the 17° C. line. This line gives us the successive vapour pressures in mms. of mercury which must exist at 17° C. for various differences of wet and dry bulb readings. Referring to the column for a difference of 3° C., we find that the vapour pressure at 17° C. is 10·1 mm. of mercury.

We now turn to Table VI., which gives us the *maximum* vapour pressures possible at different temperatures, *i.e.* the vapour pressures when the air is saturated with water vapour. It will be obvious that the dew-point for the particular example in question will be that temperature to which the air at 17° C. must be cooled in order that 10·1 mm. may be the maximum vapour pressure. We see from the table that the result lies between 11°·4 C. and 11°·6 C., and a simple calculation in proportion will show that the required temperature is 11°·51 C.

Note.—In cases when the difference between the wet and dry bulb readings comes to a decimal of a degree, as, for example, 2°·3, then the vapour pressure must be found by a simple calculation in proportion. Thus for a dry-bulb reading of 15° C. the required value lies between 9·9 and 8·6, and will be equal to $9{\cdot}9 - 0{\cdot}3\,(9{\cdot}9 - 8{\cdot}6) = 9{\cdot}5$ mm. of mercury.

Relative Humidity.—A knowledge of the dew-point gives us some idea of the condition of dampness or dryness of the atmosphere, for if the dew-point is several degrees below the normal temperature, we at once infer that the air is very dry, whilst on the other hand, if very little cooling is required before the saturation point is reached, the air must be very damp.

A more direct measurement of the hygrometric state of the atmosphere, as it is called, is given by a comparison of the amount of water vapour present in a given volume of air with the maximum amount it could hold, *i.e.* the amount which would be held by the air in a state of saturation. This ratio is spoken of as the **relative humidity,** or the fractional saturation, and is expressed either as a decimal or as a percentage.

We therefore have (when the air is at $t°$ C.),

$$\text{relative humidity} = \frac{\text{mass of water vapour present in 1 c.c. of air at } t° \text{ C.}}{\text{mass of water vapour to saturate 1 c.c. of air at } t° \text{ C.}}$$

But the ratio of the masses equals the ratio of the densities,

$$\therefore \text{ relative humidity} = \frac{\text{density of water vapour present in 1 c.c. of air at } t^\circ \text{ C.}}{\text{density of water vapour to saturate 1 c.c. of air at } t^\circ \text{ C.}}$$

But the density is proportional to the pressure,

$$\therefore \text{ relative humidity} = \frac{\text{pressure of water vapour present in 1 c.c. of air at } t^\circ \text{ C.}}{\text{pressure of water vapour to saturate 1 c.c. of air at } t^\circ \text{ C.}}$$

Finally, it will be remembered that the normal vapour pressure exerted by the vapour present in any given sample of air at t° C. becomes the saturation pressure at the corresponding dew-point.

Hence we have finally that

$$\textbf{relative humidity} = \frac{\textbf{saturation pressure at the dew-point}}{\textbf{saturation pressure at the normal temp.}}$$

Hence, to determine the relative humidity, it is merely necessary to know the normal temperature and the corresponding dew-point, and to determine from Table VI. the corresponding maximum vapour pressures; the ratio of these gives the value required.

Any of the methods already described for the determination of the dew-point may therefore be used for finding the relative humidity. The only additional method which calls for special comment is that performed with a piece of apparatus known as the chemical hygrometer.

The Chemical Hygrometer.—By means of this extremely simple apparatus, the amount of moisture actually present in a given volume of air can be directly measured. A and B (Fig. 67) are tubes containing a drying agent such as calcium chloride or phosphorus pentoxide; these are connected with an aspirator D. When the tap is turned on, water is drawn off, and its place is taken by air which is drawn in through A and B. These are carefully weighed before and after the experiment, and the difference gives the amount of moisture present in the volume of air equal to the volume run out. The experiment is now repeated,

the air this time being caused to be saturated before entering the drying tubes. This is done by passing it through tubes containing soaked wool. The ratio of the masses gives the relative humidity.

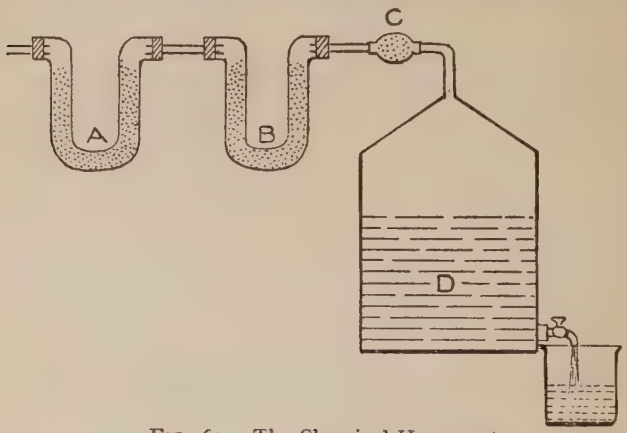

FIG. 67.—The Chemical Hygrometer.

This is an accurate but very tedious method, and is on this account but little used.

Example 2.—*Find the relative humidity from the data supplied in worked Example 1 given above.*

Solution.—Temperature of atmosphere = 17° C.
Temperature of dew-point = 11°·51 C.

From Table VI. we have

Saturation pressure at the normal temperature = 14·395 mm.
Saturation pressure at 11°·4 C. = 10·028 mm.
Saturation pressure at 11°·6 C. = 10·161 mm.

Hence, in proportion,

Saturation pressure at dew-point (11°·51 C.) = 10·095 mm.

$$\text{Relative humidity} = \frac{\text{saturation pressure at dew-point}}{\text{saturation pressure at normal temp.}}$$

$$= \frac{10{\cdot}095}{14{\cdot}395} = \mathbf{0{\cdot}701},$$

Example 3.—*A cubic metre of air at 15° C., just saturated with water vapour, meets and mixes with another equal volume of air at 13° C., also saturated with water vapour. Calculate the excess of water held in the mixture above that required to saturate air at 14° C.*

Solution.—

Saturation pressure at 15° C. = 12·674 mm.
Saturation pressure at 14° C. = 11·8835 mm.
Saturation pressure at 13° C. = 11·1370 mm.
} From Table VI.

$\therefore$ average pressures of 13° C. and 15° C. $= \frac{12\cdot674 + 11\cdot137}{2}$

$= \frac{23\cdot811}{2} = 11\cdot9055$ mm.

$\therefore$ excess of pressure of mixture above saturation pressure at 14° C. $= 11\cdot9055 - 11\cdot8835$

$= 0\cdot0220$ mm.

Hence condensation must occur equal to a mass of vapour whose volume is 2 cubic metres and whose pressure is 0·0220 mm.

EXAMPLES ON CHAPTER IX

1. Define the dew-point, and state how it can be found experimentally. What experimental errors is your method liable to ?
2. Explain (*a*) why icebergs are usually surrounded by fog, and (*b*) why a morning fog usually disappears before noon.
3. On a day when the barometer is 760 mm. high, the temperature of the air is 20° C., and the relative humidity is 0·5 ; what fraction of the whole pressure of the air is due to water vapour ? The saturation pressure at 20° C. is 18 mm.
4. Define the relative humidity of the atmosphere, and show in a general way how the readings of the wet and dry bulb hygrometer are connected with it. On a clear evening the dew-point is 42° F. Explain the effect of the moisture present upon the fall of temperature later, and consider the probability or otherwise of frost.
5. Why may the barometer be expected to rise in hard dry weather ? Give some explanation of the formation of hoar frost, snow, and hail respectively.

6. Explain why the readings of the wet and dry bulb thermometers are generally different. Give an account of the formation of dew and mist. If the vapour pressure of water at 15° C. is 12·7 mm. and that at a temperature of 12° C. is 10·5 mm., find the humidity of the air when the temperature is 15° C., and the dew-point is 12° C.
7. 100 c.c. of oxygen saturated with water, are collected at a pressure of 40 mm. and 15° C. Find the volume of dry oxygen at 0° C. and 760 mm. pressure, given that the maximum vapour pressure of aqueous vapour at 15° C. is 12·7 mm.
8. The dew-point of the air in a greenhouse rises from 9°·5 C. to 20°·2 C. ; calculate the proportion in which the water vapour present is increased.
9. Explain why a mixture of two different streams of air at different temperatures may give rise to a mist or cloud. How does this occur in nature ?
10. Draw an apparatus for ascertaining the dew-point, and describe carefully the mode of making observations. Would you expect the dew-point to be at once affected if a sudden fall in the barometer occurred ? If so, say how and why ?
11. One day when the room temperature was 84° F. an experiment showed a deposit of dew at 57° F. What was the relative humidity of the air in the room ?
12. A wet and dry bulb hygrometer showed a dry-bulb reading of 21°·4 C. and a wet-bulb reading of 18°·9 C. Comment in detail on the state of the atmosphere in the light of this data.
13. The dew-point of air in a room whose temperature was 16° C. was found to be 7°·0 C., and the barometer stood at 75·82 cms. If the density of dry air at N.T.P. is 0·001293 grams per c.c., calculate the density of the air in the room (apart from the water vapour present).
14. On drawing 27 litres of air through a chemical hygrometer the weight of the drying tubes was found to have increased by 0·288 gram. The air temperature was 18° C. Find (*a*) the dew-point, and (*b*) the relative humidity.
15. The weight of water vapour present in unit volume of air is known as the absolute humidity. Find this for a sample of air whose temperature is 14° C. when the dew-point is found to be 8°·8 C., given that the density of water vapour at N.T.P. is 0·0008064 gram per c.c.

CHAPTER X

SPECIFIC HEAT

Distinction between Quantity of Heat and Temperature.—Thus far we have confined our study of heat to those aspects of the subject which are concerned with changes of temperature, *i.e.* with the intensity of hotness. We have now to see how this is related to the idea of quantity of heat. The two are very different, and it is possible for a small amount of heat to produce big temperatures, *i.e.* much hotness. When we heat water in two saucepans, one large and the other small, if the rate of heat is the same, the water in the smaller saucepan will be quite hot before the water in the larger saucepan is properly warm.

The Heating of Different Substances.—Let us next make a different kind of comparison. Take two saucepans of the same size, fill one with water and another with some other liquid—for example, alcohol. If these are heated at equal rates, we can compare, by dipping a thermometer in each, the temperatures reached after any given time, and we find that the alcohol gets hot more quickly than the water. Practically speaking, no two substances of the same mass get hot at the same rate. The same applies to the rate at which substances lose heat on cooling. The following experiment shows this very clearly in the case of solids. Take four small metal balls, say of iron, copper, zinc, and lead, all of the same size. Keep them for a time in a hot oil-bath until all have the same temperature, *i.e.* the temperature of the oil-bath. Now place the hot iron ball on a thick slab of paraffin wax. As the ball cools it gives heat to the wax slab, which melts and so allows the ball to sink down. Do this for each ball in turn. It is found that each sinks to a different depth, showing that different quantities of heat have been given out in cooling. The iron sinks the deepest, and the lead the least.

Hence they must have taken in or absorbed different quantities of heat when heated in the oil-bath, although they both attained the same temperature. We sometimes express this fact that different substances require different quantities of heat to raise them through the same temperature by saying that they have different **capacities for heat,** or different **thermal capacity.**

The Unit of Heat.—If we are to measure quantities of heat we must have a unit. The most convenient method is to measure the rate at which some substance increases in temperature. Because, however, this rate is different for different substances, we must choose a standard material. The material chosen is cold water. *The unit quantity of heat in the metric system is that amount of heat which must be given to* 1 *gram of water at* 4° *C. in order to raise its temperature by* 1° *C.* We call this unit the **calorie.** *In the British system of units the unit of heat is that amount which must be supplied to* 1 *pound of water at* 39° *F. in order to raise its temperature* 1°. This unit is called the **British Thermal Unit** (abbreviated to **B.T.U.**).

Specific Heat.—It is a curious fact that water requires more heat to raise its temperature by any given amount than any other substance whatever. It is possible to measure accurately the amount of heat required to raise the temperature of unit quantity of a substance by 1°. We call this quantity of heat the **specific heat** of the given substance. Generally speaking, the specific heat of a liquid is much greater than that of a solid, *i.e.* solids get hot much more quickly than liquids; whilst the specific heats of the metals are low as compared with the non-metals. *The specific heat of water is, of course, unity.*

The specific heats of a few of the commoner substances are shown in Table VII. at the end of the book. A few examples will help to appreciate the importance of specific heat.

Suppose a 5-lb. brick is used as a foot-warmer. If its temperature was originally 200° F., and it cools down to 100° F., since the specific heat of brick is 0·2, every degree F. of cooling means that the brick has given out $5 \times 0{\cdot}2$, *i.e.* 1 B.T.U., of heat; so that the total amount of heat given out by the foot-warmer in cooling 100°F. is 100 B.T.U.'s. Now suppose we have a foot-warmer containing 5 lb. of water at 200° F.; the specific heat of water is 1, and therefore in cooling 100° the water gives up 500 B.T.U.'s of

heat. Water is therefore a much better agent for foot-warmers, bed-warmers, etc., than brick. Again, when the sun is shining equally on land and sea, owing to the higher specific heat of the latter, the temperature of the land increases faster than that of the sea. At night-time the converse is true, and we shall see from Chapter XIII. that this comparison between land and sea is intensified by the property of conduction. Water tends to retain heat much longer than earth.

Thermal Capacity.—In the term specific heat we are confined to 1 gram or 1 lb. of any given substance. If we denote by c the value of the specific heat, then for 2 grams the amount of heat required to raise the temperature one degree is $2c$ calories, and, generally, the amount of heat required to raise the temperature of a substance weighing m grams by 1 degree Centigrade will equal mc calories. We call this quantity the **thermal capacity** or **the capacity for heat** of the substance. Thus the thermal capacity of a body weighing 5000 grams, and the specific heat of whose material is 0·09, will be $5000 \times 0{\cdot}09$, *i.e.* 450 calories; whilst in the British system the thermal capacity of a 5-lb. brick (specific heat $= 0{\cdot}2$) $= 5 \times 0{\cdot}2 = 1$ B.T.U.

Heat absorbed or emitted during a Change in Temperature.—We are now in a position to calculate the amount of heat emitted or absorbed by a body during any change of temperature.

Let m = mass of the body.
,, c = specific heat of the body.

Then for every degree change in the temperature of the body, mc calories are lost or absorbed according as the temperature has fallen or risen.

Let θ = initial temperature of the body,
and ϕ = final temperature of the body.

Then the total change in temperature $= \theta - \phi$, assuming a cooling to have taken place.

$\therefore$ Total heat lost $= mc(\theta - \phi)$ calories,

i.e. **Heat lost (or gained) = mass of body × its specific heat × its change in temperature.**

Example 1.—*How many calories are there in* 1 *B.T.U.?*

Solution.—This is equivalent to the number of **calories** required to raise the temperature of 453·6 grams of water $\frac{5}{9}$° C., since 453·6 grams = 1 lb. and $\frac{5}{9}$° C. = 1° F.

∴ Number of calories required = mass of water × its specific heat × rise in temperature

$= 453{\cdot}6 \times 1 \times \frac{5}{9}$

= 252 calories = **number of calories per B.T.U.**

Example 2.—*How much heat is required to raise the temperature of a copper cylinder weighing* 50 *grams by* 40° *C.? The specific heat of copper may be taken as* 0·095.

Solution.—It requires 0·095 calories to raise the temperature of 1 gram of copper 1° C.

∴ it requires (0·095 × 50) to raise the temperature of 50 grams of copper 1° C.

∴ it requires (0·095 × 50 × 40) calories to raise the temperature of 50 grams of copper 40° C.

Hence heat required = mass of cylinder × its specific heat × its rise in temperature $= 0{\cdot}095 \times 50 \times 40$

= 190·0 calories.

Example 3.—*How much heat must be supplied per hour by the radiators of a school assembly hall if there are* 300 *scholars, for each of whom there must be supplied* $\frac{1}{3}$ *cubic foot of fresh air per second for efficient ventilation, if the outside temperature is* 32° *F., and the room is to be maintained at a steady temperature of* 65° *F.* (1 *cubic foot of fresh air weighs* 1·3 *oz., and the specific heat of air is* 0·23.)

Solution.—Heat required to raise the temperature of 1 lb. of air 1° F. = 0·23 B.T.U.'s.

Weight of air required per second per boy

$$= \tfrac{1}{3} \times 1{\cdot}3 \text{ oz.} = \tfrac{1}{3} \times \frac{1{\cdot}3}{16} \text{ lb.}$$

∴ Total of air required per sec., $= \frac{1}{3} \times \frac{1{\cdot}3}{16} \times 300$ lb.

$\therefore$ Heat required to raise the temperature of the hall per

$$\text{second} = \left(\tfrac{1}{3} \times \frac{1 \cdot 3}{16} \times 300\right) \times 0 \cdot 23 \text{ B.T.U.'s.}$$

But total rise in temperature required for the fresh air

$$= 65 - 32 = 33^\circ \text{ F.}$$

$\therefore$ Total heat required per second

$$= \left(\tfrac{1}{3} \times \frac{1 \cdot 3}{16} \times 300 \times 0 \cdot 23 \times 33\right) \text{ B.T.U.'s.}$$

$\therefore$ Total heat required per hour

$$= \tfrac{1}{3} \times \frac{1 \cdot 3}{16} \times 300 \times 0 \cdot 23 \times 33 \times 3600 \text{ B.T.U.'s}$$

$$= \mathbf{220{,}000 \text{ B.T.U.'s.}}$$

Example 4.—*The commercial unit of heat-supply is the* **therm,** *and is equal to* 100,000 *B.T.U.'s. What is the cost of heating the assembly hall per hour in the above example at* 1*s.* 4*d. per therm?*

Solution.—Number of therms per hour $= \frac{220{,}000}{100{,}000} = 2 \cdot 2.$

$\therefore$ Cost per hour $= 2 \cdot 2 \times$ 1s. 4d. $= 35 \cdot 2$ pence

$= \mathbf{2s.\ 11\tfrac{1}{4}d.}$

Mixtures of Different Substances at Different Temperatures.—We pointed out in Chapter VI. that it was Joseph Black who first drew attention to the phenomenon of latent heat. Black did more than this, however. For it was this discovery that prompted him to inquire **how much heat** was required to change the state of a substance. In other words, it opened up the whole subject of the **quantitative** study of heat, and led to the conception of the calorie as the unit of quantity of heat, and to the idea of and definition of specific heat. In order to determine experimentally the specific heat of a substance, Black invented what is called the "method of mixtures." When a hot substance is mixed with a cold substance, something in the nature of a "flow of heat" takes place from the hotter to the cooler body, the latter being warmed and the former being cooled until both attain a common temperature intermediate between the two original temperatures. Assuming that no heat has been lost

to the surroundings, we may express the behaviour by the statement:

Heat lost by the hotter body = heat gained by the cooler body.

We may now apply the results of the previous paragraphs.

Mass of hot body × its specific heat × its fall in temperature = mass of cold body × its specific heat × its rise in temperature.

Let M = mass of hot body.
" C = its specific heat.
" θ = initial temperature of hot body.
" m = mass of cold body.
" c = its specific heat.
" ϕ = initial temperature of cold body.
" t = final common temperature of the mixture.

Then we have

$$MC(\theta - t) = mc(t - \phi)$$

and given any six of these quantities, the seventh may be at once calculated.

Example 5.—*Twenty-five grams of hot copper are added to 120 grams of water at 12° C., and the temperature of the mixture becomes 15° C. Assuming that no heat losses are incurred during this process, what was the original temperature of the copper?* (*The specific heat of copper* = 0·094.)

Solution.—Heat lost by hot copper = heat gained by cold water.

∴ Mass of hot copper × its specific heat × its fall in temperature = mass of cold water × its specific heat × its rise in temperature.

Let t = original temperature of the hot copper.

$$\therefore \quad 25 \times 0{\cdot}094 \times (t - 15) = 120 \times 1 \times (15 - 12).$$

$$\therefore \quad (t - 15) = \frac{120 \times 3}{25 \times 0{\cdot}094} = 153{\cdot}2.$$

$$\therefore \quad t = 153{\cdot}2 + 15$$

$$= 168°{\cdot}2 \text{ C.}$$

Calorimetry—The Water Equivalent.—If we try to experiment on the lines of the above paragraph, we find that certain difficulties occur. For example, it is impossible to mix together

two solids at different temperatures, even in powdered form, without losing much of the heat to the surroundings, or with any hope of a uniform distribution of temperature throughout the body of the substance. With two liquids the problem is considerably simplified. We may **quickly** pour the hot liquid into the vessel containing the cold liquid. Such a vessel in which heat measurements are made is spoken of as a **calorimeter.** The reason for *quickly* pouring the hot liquid into the calorimeter is to avoid losses to the surroundings during the transference.

By quickly stirring the mixture with a stirrer (in the absence of a special stirrer, the thermometer may be used) we cause the heat to be uniformly distributed throughout the body of the mixture, and so eliminate a further source of error.

It will be obvious that the presence of the calorimeter must modify the calculation involved, for when the hot liquid is introduced, not only is heat absorbed by the cold liquid, but also by the cold calorimeter. The equation now becomes :

Heat lost by hot liquid = heat gained by cold liquid
+ heat gained by cold calorimeter.

Let m_1 = mass of calorimeter.
,, c_1 = specific heat of the material of the calorimeter.

Its initial temperature is, of course, the same as that of the cold liquid, *i.e.* ϕ.

Then we now have

$$\begin{aligned} \mathrm{MC}(\theta - t) &= mc(t-\phi) + m_1c_1(t-\phi) \\ &= (mc + m_1c_1)(t-\phi), \text{ by algebra.} \end{aligned}$$

Comparing this with the equation on page 126, it will be seen that the only difference made by the calorimeter is in the addition of the term m_1c_1 to the term mc.

We call m_1c_1 the **water equivalent** of the calorimeter, because in nearly all calorimetric experiments the cold liquid used is usually water, and the effect is just the same as if no calorimeter were considered, but instead an *additional* quantity of m_1c_1 grams of cold water were used.

For accurate work, since the thermometer and stirrer also absorb heat from the hot liquid, their water equivalent should also be added.

It is desirable that the calorimeter should take up the common temperature of its contents quickly, and accordingly this vessel is usually made of a good conducting material (see Chapter XIII.), such as copper. The final temperature of the mixture is not, however, attained instantaneously, and if the copper calorimeter is resting on the laboratory bench, some heat will be lost during this period to the surroundings. To prevent this as far as possible, the calorimeter should be well polished (see Chapter XIV.), and should be protected by "jackets" of materials which are bad conductors—*e.g.* an air-jacket AA (Fig. 68) and a water-jacket BB. The base of the calorimeter is protected from BB by three corks.

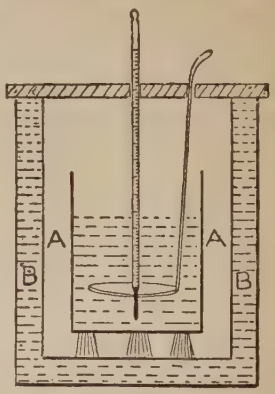

FIG. 68.—Calorimeter and Jackets.

Mixtures of Solids and Liquids—Heaters.—When a solid and a liquid are mixed at different temperatures, we usually arrange for the solid to be at the higher temperature. The transference must be rapid to avoid losses, and the heaters in which the solids are warmed are designed with this object in view. Fig. 69 shows one form suitable for a substance which can be experimented upon in the form of broken pieces, such as marble or lead shot. It consists of a metal vessel A containing water, closed by means of a stopper C

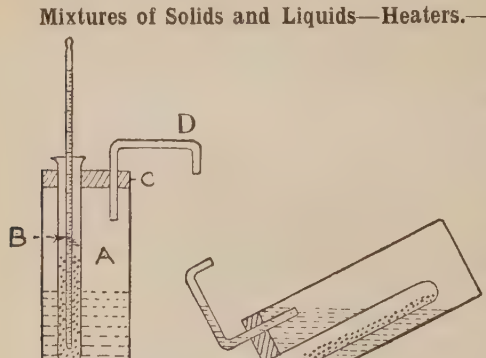

FIG. 69.—Heater suitable for Bits of Marble or Lead Shot.

fitted with two holes, one large and one small. Into the larger hole is fitted a tube B containing a thermometer embedded in the substance to be experimented upon. D is an outlet for steam. The water in A is heated until the thermometer in B has recorded a steady temperature for some time, and the apparatus is then tilted over the calorimeter and the contents of B are quickly poured out. It will be seen from Fig. 69, *b*, that this may be done without fear of the water flowing out, provided too much has not been introduced initially. When the solid is

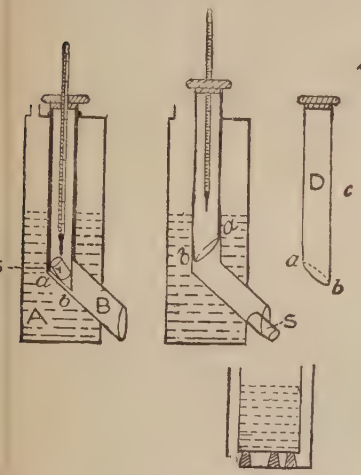

FIG. 70.—Heater for Solid Body.

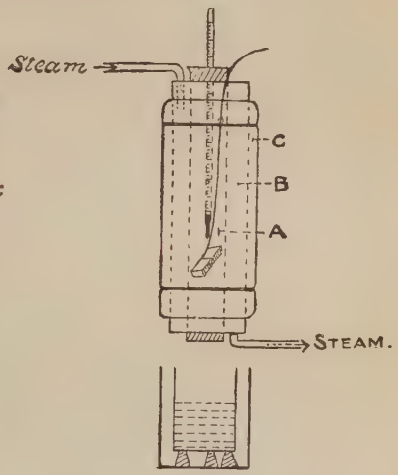

FIG. 71.—Steam Heater.

in one piece, the heater may take the form indicated in Fig. 70. A vessel A containing water acts as a jacket to a tube B running down the middle of it; the lower extremity of B is bent as shown, and emerges at the side of A, the end being open. Fitting exactly into B is another cylindrical tube D (see Fig. 70, *c*). Its upper end is closed by a stopper through which passes a thermometer, and its lower end *ab* is so shaped as to be able to fit the sloping part of B, thus converting it into a closed tube (Fig. 70, *a*). This enables us to introduce a solid S and to heat it to a constant temperature by boiling the water in A. When the temperature

has remained steady for some time, the tube D is rotated by means of the stopper into the position shown in Fig. 70, *b*, and the solid is now able to slip down the sloping end of B into the calorimeter.

Fig. 71 illustrates a better form of heater, of a type fitted in many good laboratories. A long brass cylindrical chamber A is jacketed by another brass chamber B, and this in turn is surrounded by a packing C of some non-conducting material like felt or wool. The solid is suspended in A by means of a thread, and steam is passed through B as shown until a steady temperature is indicated by the thermometer. The solid is then quickly lowered into the calorimeter.

Example 6.—*A mass of iron weighing* 450 *grams is taken out of ice-cold water and immersed in an iron vessel weighing* 90 *grams, and containing* 200 *grams of water at* 13° *C. To what temperature is the water reduced if the specific heat of iron is* $\frac{1}{9}$ *?*

Solution.—The calorimeter is here of iron, and its water equivalent is therefore equal to the mass of iron vessel × its specific heat.

$\therefore$ Water equivalent of calorimeter $= 90 \times \frac{1}{9} = 10$ calories.

Now heat lost by warm water = heat gained by cold metal.

$\therefore$ (Mass of warm water + water equivalent of calorimeter) × its specific heat × its fall of temperature = mass of cold metal × its specific heat × its rise in temperature.

Let t = final temperature of mixture.

Also, initial temperature of cold metal = 0° C.

$$\therefore \quad (200+10)\times 1\times(13-t)=450\times\tfrac{1}{9}\times(t-0)$$

$$\therefore \quad 210(13-t)=50t.$$

$$\therefore \quad 2730-210t=50t.$$

$$\therefore \quad 2730=50t+210t$$

$$=260t.$$

$$\therefore \quad t=\frac{2730}{260}=\mathbf{10^{\circ}\cdot 5\ C.}$$

Example 7.—*In order to estimate the temperature of a furnace, a small piece of platinum weighing* 3 *oz. was kept in it for some little time, and was then quickly transferred into a platinum crucible weighing* 1·2 *oz. and containing* 10 *oz. of water at* 40° *F. The*

temperature rose to 60° F. The specific heat of platinum may be taken as 0·038. What was the temperature of the furnace?

Solution.—The water equivalent of the platinum calorimeter
= mass of calorimeter × its specific heat

$$= \left(\frac{1\cdot2}{16} \times 0\cdot038\right).$$

Now heat lost by hot platinum solid = heat gained by cold water and calorimeter.

Let t = temperature of furnace, and hence temp. of hot solid.

$\therefore$ Mass of hot solid × its specific heat × its fall of temperature
= (mass of cold water + water equivalent of calorimeter) × rise in temperature.

$$\therefore \frac{3}{16} \times 0\cdot038 \times (t-60) = \left\{\frac{10}{16} + \left(\frac{1\cdot2}{16} \times 0\cdot038\right)\right\} \times 1 \times (60-40)$$

$$= 0\cdot6535 \times 20.$$

$$\therefore \quad t-60 = \frac{0\cdot6535 \times 20 \times 16}{3 \times 0\cdot038}$$

$$= 1835.$$

$$\therefore \quad t = 1835 + 60$$

$$= \mathbf{1895^\circ\ F.}$$

Testing the Thermal Value of the Domestic Gas-Supply.—Until very recently Gas Companies charged the consumer by volume—so much per 1000 cubic feet of gas passing through the meter, and indeed the meters are still unchanged in this respect. Owing, however, to the varying qualities of the gas-supply from the heat point of view, Gas Companies are now compelled by Act of Parliament to state the number of therms (1 therm = 100,000 B.T.U.'s) of heat per 1000 cubic feet of gas, and the consumer is now charged by the therm instead of by the volume.

It is possible to apply a sufficiently accurate test of the heat value of a local gas-supply (where the small dial of the gas meter can be read in fifths or less of a cubic foot) by the following method. Weigh an empty kettle by means of a spring balance, fill it with water, and weigh again. Next observe the temperature of the water by means of a thermometer, and then place the kettle on a good gas-ring, surrounding the burner with sheets of asbestos to

ensure that as far as possible the gas is all utilised in heating the kettle and its contents. Take a reading of the gas meter, and turn on the gas. Note the time taken to reach the boiling-point, and read the gas meter again, the difference in the two readings giving the number of cubic feet of gas used.

The calculation involved is best shown by quoting the results of an actual experiment as follows :

Weight of empty kettle $= 2{\cdot}5$ lb.
Weight of kettle + water $= 10$ lb.
$\therefore$ Weight of water alone $= 7{\cdot}5$ lb.
Temperature of cold water $= 56°$ F.
Final temperature of water $= 212°$ F.
Volume of gas used $= 2{\cdot}8$ cubic feet.
$\therefore$ Heat required by the water = mass of water $\times$ its specific heat $\times$ rise in temperature

$$= 7{\cdot}5 \times 1 \times (212 - 56)$$
$$= 1170 \text{ B.T.U.'s.}$$

This is supplied by 2·8 cubic feet of gas.

$\therefore$ 1000 cubic feet of gas furnishes $\frac{1170 \times 1000}{2{\cdot}8}$

$$= 417{,}860 \text{ B.T.U.'s.}$$
$$= \mathbf{4{\cdot}17 \text{ therms.}}$$

This, it should be noted, is the **effective** thermal capacity of gas. The actual thermal capacity must be higher, since some of the heat is utilised in heating the kettle (and can be allowed for by including its water equivalent in the calculation), and some is lost by radiation. It is for the correction of this latter loss that the duration of the experiment should be noted.

Newton's Law of Cooling.—In spite of the various precautions taken to prevent heat losses to the surroundings during calorimetric experiments, some loss is unavoidable, and corrections are therefore necessary where accurate results are required. These are usually based on **Newton's Law of Cooling,** which states that a hot body cools **at a rate which is proportional to the excess of its temperature over that of its surroundings.**

Fig. 72 shows two " cooling curves " for water, connecting temperature with time. The same quantity was used in each case,

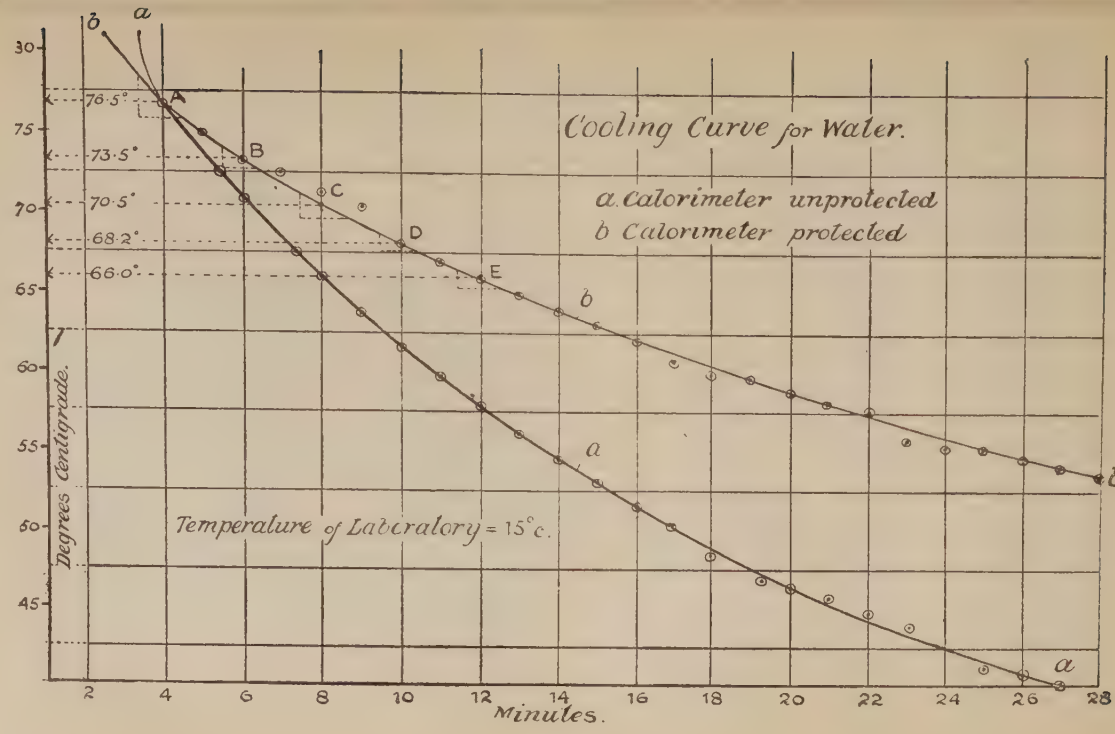
Cooling Curve for Water.
a Calorimeter unprotected
b Calorimeter protected
Temperature of Laboratory = 15°c.
Degrees Centigrade.
Minutes.
76·5°
73·5°
70·5°
68·2°
66·0°
A
B
C
D
E
a
b
30
75
70
65
60
55
50
45
2
4
6
8
10
12
14
16
18
20
22
24
26
28

FIG. 72.

but in the one case a bare unprotected calorimeter was used, whilst in the other the calorimeter was surrounded by jackets as shown in Fig. 73. The curves show strikingly the importance of suitably protecting the calorimeter. The temperature of the laboratory was 15° C., and the following table shows how Newton's Law was tested at various points on the curve *b*:

Point on Curve.	Temperature, ° C.	Excess over 15° C.	Average Rate of Cooling, ° C. per min.	Ratio = Rate of Cooling / Excess over 15° C.
A	76·5	61·5	2·2	$\frac{2\cdot2}{61\cdot5}=0\cdot036$
B	73·5	58·5	1·5	$\frac{1\cdot5}{58\cdot5}=0\cdot025$
C	70·5	55·5	1·4	$\frac{1\cdot4}{55\cdot5}=0\cdot02_5$
D	68·2	53·2	1·1	$\frac{1\cdot1}{53\cdot2}=0\cdot021$
E	66·0	51·0	1·0	$\frac{1\cdot0}{51\cdot0}=0\cdot0199$

The fourth column is obtained by considering the interval of time of which the point in question is the middle. Thus for B, the interval is from the 5½ minutes mark to the 6½ minutes mark, and the cooling for this period is 1·5° C.; and so for the other points.

It will be seen that Newton's Law is not strictly true, the ratio $\frac{\text{rate of cooling}}{\text{excess over surroundings}}$ tending to diminish as the temperature decreases. As a matter of fact, Newton's Law is an approximation to a more correct one known as Stefan's Law of Radiation, but within small ranges of temperature cooling Newton's Law is sufficiently accurate for ordinary purposes.

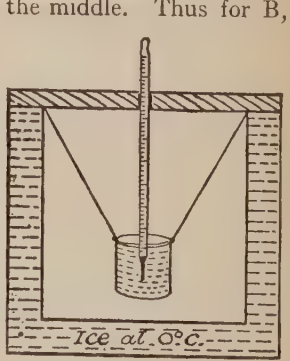

FIG. 73.

Specific Heat of Liquids by Method of Cooling.—This is an application of Newton's Law of Cooling. Heat some liquid in a calorimeter up to, say, 100° C., and then suspend it in an enclosure (Fig. 73) and let it cool. We have already seen (Fig. 72) that at any given temperature θ, the rate of decrease of temperature per second can be determined from the temperature-time curve.

Let m = mass of the liquid, and c = its specific heat.
,, m_1 = mass of the calorimeter and c_1 = its specific heat.
,, θ_1 = the rate of fall of the temperature per second at the temperature θ.

Then the heat lost per second $= (mc + m_1c_1)\theta_1$ calories.

This quantity is independent of the nature of the liquid; it depends only upon the temperature and upon the weight of the liquid and calorimeter.

Now repeat the experiment, using in the calorimeter the liquid whose specific heat is required, **so that the liquid cools under the same conditions as regards area of cooling surface and excess temperature.**

Let M = its mass, and C = its specific heat.

Draw up a cooling curve, and determine from it the rate of fall θ_2 of the temperature per second at the temperature θ.

Then the heat lost per second $= (MC + m_1c_1)\theta_2$.

Hence we have $(MC + m_1c_1)\theta_2 = (mc + m_1c_1)\theta_1$,

$$\text{whence } \frac{MC + m_1c_1}{mc + m_1c_1} = \frac{\theta_1}{\theta_2}$$

from which C may be determined.

In the actual experiment it is usual to determine the time taken for the temperature to fall a definite small amount; for example, 1° between, say, 30° and 29° in each case. The rate of fall will then be equal to $\frac{\text{fall in temperature}}{\text{time of fall}}$

$$i.e. \frac{1}{\text{time of fall from } 30° \text{ to } 29°}$$

Hence we have

$$\frac{MC + m_1c_1}{mc + m_1c_1} = \frac{\text{time of fall from } 30° \text{ to } 29° \text{ for water}}{\text{time of fall from } 30° \text{ to } 29° \text{ for liquid}}$$

This saves plotting a curve, and is therefore very convenient. The result is checked by taking the time of fall at two or three different temperatures.

Example 8.—*A calorimeter is filled with 20 grams of water, and is observed to take 64 seconds to cool from 72° C. to 70° C. The calorimeter is now emptied and filled with 25 grams of a hot liquid which is observed to take 56 seconds to cool through the same range of temperatures. Neglecting the heat taken up by the calorimeter, what is the specific heat of the liquid?*

Solution.—Let c = specific heat of the liquid. Since conditions of cooling are the same, we have

$$\frac{\text{mass of 1st liquid} \times \text{its specific heat}}{\text{mass of 2nd liquid} \times \text{its specific heat}} = \frac{\text{time 1st liquid cools 2° C.}}{\text{time 2nd liquid cools 2° C.}}$$

$$\therefore \quad \frac{20 \times 1}{25 \times c} = \frac{64}{56}$$

$$\therefore \quad c = \frac{56 \times 20}{25 \times 64} = 0{\cdot}7.$$

Example 9.—*What would have been the result in the above example if the weight of the copper calorimeter (15 grams) were taken into account?*

Solution.—Specific heat of copper = 0·094.

$$\therefore \quad \frac{(\text{mass of water} \times \text{its sp. heat}) + (\text{mass of calorimeter} \times \text{its sp. heat})}{(\text{mass of liquid} \times \text{its sp. heat}) + (\text{mass of calorimeter} \times \text{its sp. heat})} = \frac{\text{time 1st liquid cools 2° C.}}{\text{time 2nd liquid cools 2° C.}}$$

$$\therefore \quad \frac{(20 \times 1 + (15 \times 0{\cdot}094)}{(25 \times c) + (15 \times 0{\cdot}094)} = \frac{64}{56}$$

$$\therefore \quad 64(25c + 1{\cdot}41) = 56 \times 21{\cdot}41.$$

$$\therefore \quad 1600c + 90{\cdot}24 = 1198{\cdot}96.$$

$$\therefore \quad 1600c = 1108{\cdot}72.$$

$$\therefore \quad c = 0\,693,$$

which shows that no very serious error is incurred in omitting the calorimeter from the calculation.

Specific Heats of Gases—The Two Special Cases.—When we turn our attention to the determination of the specific heats of gases, we are confronted with the fact that for this group of substances pressure- and volume-changes with temperature are very marked, and as a result the specific heat of a gas is different for every new set of conditions of pressure, volume, and temperature. Accordingly, the term "specific heat" can have little value without restricting the gas to some qualifying set of conditions.

The conditions which are found to give values of great importance are:

(*a*) Constant volume.

(*b*) Constant pressure.

These we shall consider in turn.

Specific Heat of a Gas at Constant Volume—Method of Mixtures. It is possible to use the method of mixtures by enclosing the gas under high pressure, say, 100 atmospheres, in a little bomb. This is heated to between 200° C. and 300° C., and it is then immersed in a calorimeter containing cold water.

Let M = mass of gas, and C = its specific heat.
,, M_1 = mass of bomb, and C_1 = its specific heat.
,, m = mass of water, and c = its specific heat.
,, m_1 = mass of calorimeter, and c_1 = its specific heat.
,, θ = initial temperature of the water in the calorimeter.
,, φ = initial temperature of the bomb and gas.
,, t = resulting temperature of the mixture.

Then we have

$$(MC + M_1C_1)(\varphi - t) = (mc + m_1c_1)(t - \theta),$$

from which C may be determined. A preliminary experiment with the bomb evacuated is, of course, first necessary in order to determine the water equivalent of the bomb. This is not an accurate method, owing to the fact that even at 100 atmospheres pressure the mass of the gas is very small compared with the rest of the apparatus; and as its specific heat is also small, the quantity of heat it absorbs or loses is very small compared with that of the bomb and calorimeter, so that errors in the readings affect the result very seriously.

A much more accurate method, based on latent heat determination, is given in the next chapter.

It is usual to denote the specific heat of a gas at constant volume by c_v, and that at constant pressure by C_p.

Determination of C_p—Regnault's Experiment.—A method due to Regnault is illustrated in Fig. 74. The gas is stored in a reservoir A, and is led along a copper tube into an oil-bath B. The gas-supply is regulated by means of a conical screw R, and its pressure is indicated by the manometer M. The oil-bath B is maintained at a high temperature, and the gas is brought up to this temperature by being sent round a long spiral coil. It emerges from the oil-bath, and is passed through the calorimeter C, and is then allowed to escape into the air. The stream of hot gas will gradually raise the temperature of the calorimeter and its contents.

Let M = the mass of gas passed through the calorimeter during the experiment.

,, c_p = its specific heat at constant pressure.

,, t = the temperature of the oil-bath B, *i.e.* the temperature at which the gas enters the calorimeter.

,, t_1 = the initial temperature of the calorimeter and its contents.

,, t_2 = the final temperature of the calorimeter and its contents.

,, m = the mass of water in the calorimeter.

,, c = its specific heat.

,, m_1 = mass of calorimeter.

,, c_1 = its specific heat.

Then the total heat absorbed from the gas by the calorimeter and contents

$$= (mc + m_1c_1)(t_2 - t_1) \text{ calories.}$$

Now the gas always issues into the air at the temperature of the calorimeter. Therefore at the beginning of the experiment the cooling of the gas is from t° to t_1°, *i.e.* $(t - t_1)$, whilst at the end it is from t° to t_2°, *i.e.* $(t - t_2)$.

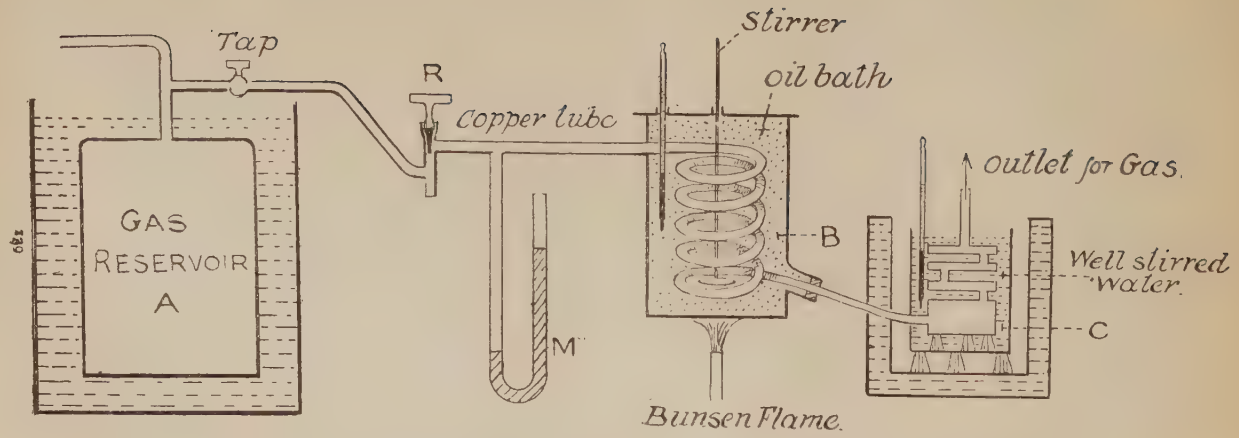

FIG. 74.—Regnault's Determination of the Specific Heat of a Gas at Constant Pressure.

Hence the **mean** cooling during the experiment

$$= \frac{t - t_1 + t - t_2}{2}$$

$$= t - \frac{t_1 + t_2}{2}$$

$\therefore$ Heat given up by the gas to the calorimeter and contents

$$= \mathrm{M}c_p\left(t - \frac{t_1 + t_2}{2}\right)$$

Hence $\mathrm{M}c_p\left(t - \frac{t_1+t_2}{2}\right) = (mc + m_1c_1)(t_2 - t_1)$,

from which c_p may be calculated.

Corrections for heat losses are applied.

This method gives fairly reliable results. The objection that the mass of the gas is small, to which reference was made in the "bomb" experiment for constant volume, does not obtain here. The gas not being stored, but being allowed to escape, we are able to pass a considerable mass through the apparatus.

Holborn and Henning, in 1907, considerably extended the utility of Regnault's method. Instead of heating the gas in an oil-bath, which necessarily gives only a small range of temperature, the gas was passed through a porcelain tube into an electrical furnace, and by this means the specific heat could be determined up to about 1400° C.

In addition to this, a further improvement was effected by attaching to the exit tube of the gas from the calorimeter a very small opening fitted with different nozzles. By this means they were able to alter the pressure in the whole apparatus up to several atmospheres. Specific heats were thus obtained under a variety of conditions of temperature and pressure. Paraffin oil was substituted for water in the calorimeter.

EXAMPLES ON CHAPTER X

1. Sketch some simple form of calorimeter, and state the use of each of its parts. Report fully on some experiment you have made with the instrument. Be careful to state the object of the experiment, and also the conclusion to which you were led by your observations.

2. What is a "heater"? What are the considerations which should guide you in your choice of one for calorimetric experiments? Describe one or two suitable forms.
3. Distinguish carefully between the temperature of a mass and the quantity of heat in it. How would you measure the quantity of heat required to raise the temperature of a copper cylinder weighing 500 grams from 15° C. to 100° C.?
4. What is meant by the thermal capacity or "water equivalent" of a body, and how is it connected with the masses and the specific heats of the various substances composing the body? How may the thermal capacity of a brass calorimeter be determined experimentally?
5. Define the term "specific heat." A calorimeter the thermal capacity of which is 12 contains 200 grams of oil of specific heat 0·5, the temperature being 10° C. 100 grams of copper (specific heat 0·1) at a temperature of −100° C. are introduced into the oil. Find the resulting temperature of the mixture.
6. The quantity of heat required to raise the temperature of a given weight of turpentine from 50° C. to 70° C. is considerably greater than the amount required to raise it from 30° C to 50° C. Describe how you would test this statement, and explain how you would satisfy yourself that the observed result could not be attributed to unavoidable errors of experiment.
7. State Newton's Law of Cooling. How could you verify it experimentally?
8. 960 grams of water at 15° C. are contained in a calorimeter whose weight is 200 grams and whose specific heat is 0·2. 500 grams of water at 65° C. are then introduced. Calculate the resulting temperature of the mixture.
9. A mass of 200 grams of copper, whose specific heat is 0·095, is heated to 100° and placed in 100 grams of alcohol at 8° C. contained in a copper vessel whose mass is 25 grams, and the temperature rises to 28°·5. Find the specific heat of alcohol.
10. How would you attempt to measure the amount of heat given out when water and sulphuric acid are mixed?
11. Distinguish between "specific heat" and capacity for heat. Describe carefully how you would determine the capacity for heat of a piece of metal.
12. A piece of metal weighing 2 lb. is known to be a mixture of iron and copper, the specific heats being respectively 0·11 and 0·086. It is heated to 100° C. and dropped into a calorimeter of total water equivalent 2·5 lb. and initial temperature 15° C. The temperature rises to 21° C. Find the weight of each ingredient.

13. A copper vessel contains 100 grams of water at 12°. When 56 grams of water at 30° are added the resulting temperature of the mixture is 18°. What is the water equivalent of the vessel? A calorimeter with water equivalent 12 contains 100 grams of water at 12°. When 100 grams of metal at 100° are added, the resulting temperature of the mixture is 20°. Find the specific heat of the metal.
14. Explain how the specific heats of two liquids may be compared by the method of cooling. What are the objections to this method?
15. Give an accurate method for the determination of the specific heat of a gas at constant pressure. How may the apparatus you describe be modified to enable the experiment to be carried out at very high temperature? How are such temperatures found to affect the result?
16. Why is it difficult to measure the specific heat of a gas by the method of mixtures? What weight of gas of specific heat 0·25 entering at 100° would require to pass through an apparatus of which the heat capacity was 50 calories per degree before raising the temperature from 15° to 17°?
17. A copper tea-kettle weighs 1½ lb. and contains 7 lb. of water at 60° F. Gas is available at a thermal value of 4 therms per 1000 cubic feet. The rate of consumption is 4 cubic feet per hour. How many cubic feet of gas are consumed by boiling the kettle and contents if the specific heat of copper is 0·094?
18. A bath is to be prepared to contain 30 gallons of water at 96° F. The temperature of the tap-water is 50° F., and the geyser can supply hot water at a steady temperature of 180° F. How much of each kind of water must be run into the bath?
19. Water and turpentine were cooled under identical conditions in a calorimeter weighing 20·3 grams (the specific heat of the material of the calorimeter being 0·1) from a temperature of 35° C. to 30° C., the respective times of cooling being 260 seconds and 92 seconds. The weight of water and turpentine used were 90·2 grams and 70 grams respectively. What was the specific heat of the turpentine?
20. The approximate temperature of a muffle furnace is determined by a calorimeter method. A piece of iron weighing ½ lb. taken from the furnace is quickly plunged into a calorimeter containing 3 lb. of water at a temperature of 59° F. The temperature of the water rises and becomes practically stationary at 80° F. Assuming that the mean specific heat of iron is 0·13, determine the furnace temperature. State three sources of error which render the calculated result an approximation only.

21. The radiator of an engine is made of brass, and weighs 60 lb. when empty; it contains 10 gallons of water. If the specific heat of brass is 0·089, and if the radiator retains all the heat it extracts from the engine, what is the rate at which its temperature will rise, assuming that it receives 40 B.T.U.'s of heat per second from the engine?
22. In order to determine the temperature of a furnace, an iron ball, of specific heat 0·112 and weight 180 grams, was heated in the furnace and then dropped into 2000 grams of water at 12° C. in a vessel needing 4·16 calories to warm it 1° C. What was the temperature of the furnace if the final temperature was 21·4° C.?
23. Define specific heat. How many B.T.U.'s of heat are absorbed per hour by the cooling system of an engine if the water enters at 60° F. and leaves at 90° F., and 5 gallons are used per minute?

CHAPTER XI

LATENT HEAT

Latent Heat of Change of State.—It has been pointed out that when a solid is converted into a liquid, or a liquid into a gas, heat must be absorbed by the substance for the change of state to be possible. Conversely, in converting a gas into a liquid or a liquid into a solid, heat must be extracted from the substance. Further, although in the one case heat is being supplied to, and in the other case extracted from, the body, its temperature during the process of the change of state remains constant, the heat energy absorbed or emitted being utilised solely for producing the change of state.

We speak of the heat utilised in changing a substance from the solid to the liquid state, or *vice versa*, as the **latent heat of fusion** of the substance; whilst the heat utilised in changing a liquid into a gas, or *vice versa*, is referred to as the **latent heat of vaporisation** of the substance.

We define the latent heat of fusion of a substance as being the amount of heat required to change 1 gram of the substance at the temperature of its melting-point from the solid state to the liquid state at the same temperature.

Similarly, **the latent heat of vaporisation of a substance is defined as the amount of heat required to change 1 gram of the substance at the temperature of its boiling-point from the liquid state to the gaseous state at the same temperature.**

The latent heat of vaporisation of a substance is usually much higher than the latent heat of fusion. The following are typical values:

Substance.	Latent Heat of Fusion (Calories).	Latent Heat of Vaporisation (Calories).
Water . .	80	537
Mercury . .	2·8	62
Sulphur . .	9·4	362
Acetic Acid . .	43	102

Table VII. at the end of the book gives the latent heat values for some of the commoner substances.

The student should notice the relatively low values for the latent heat of fusion for the elements.

Calculations involving Latent Heat Values.—Let L denote the latent heat of fusion of a solid of mass m. Then for every gram of the solid at the temperature of its melting-point, L calories are required to convert it into liquid. Hence in all mL calories must be supplied to it. Suppose, however, the solid is initially at some temperature $\phi°$ below the melting-point $\theta°$. Then in order to melt it we must first heat the body up from $\phi°$ to $\theta°$.

The heat required for this operation $= mc(\theta - \phi)$ calories, where c = the specific heat of the substance.

Hence the total heat necessary just to melt the solid (*i.e.* to convert it from a solid at $\phi°$ to a liquid at its melting-point $\theta°$

$$= mc(\theta - \phi) + mL$$
$$= m[\mathrm{L} + c(\theta - \phi)] \text{ calories.}$$

Example 1.—*Calculate the total amount of heat necessary to convert* 10 *grams of ice at* $-5°$ *C. to steam at* 115° *C., given*

that the latent heat of fusion of ice	=	80 calories.
that the latent heat of vaporisation of water	=	537 ,,
that specific heat of ice	=	0·5 ,,
that specific heat of water	=	1 ,,
and that specific heat of steam	=	0·48 ,,

Solution.—There are five stages in the operation, as follows :

(1) To convert 10 grams of ice at $-5°$ C. into ice at 0° C.
(2) To convert 10 grams of ice at 0° C. into water at 0° C.
(3) To convert 10 grams water at 0° C. into water at 100° C.
(4) To convert 10 grams of water at 100° C. into steam at 100° C.
(5) To convert 10 grams of steam at 100° C. into steam at 115° C.

(1) Heat required for Stage I. = mass of ice × its specific heat × rise in temperature
$= 10 \times 0{\cdot}5 \times 5$
$= 25$ calories.

(2) Heat required for Stage II. = mass of ice × latent heat of ice
= 10 × 80
= 800 calories.

(3) Heat required for Stage III. = mass of water × specific heat × rise in temperature
= 10 × 1 × 100
= 1000 calories.

(4) Heat required for Stage IV. = mass of water × latent heat of steam
= 10 × 537
= 5370 calories.

(5) Heat required for Stage V. = mass of steam × specific heat × rise in temperature
= 10 × 0·48 × 15
= 72 calories.

∴ Total heat required = 25 + 800 + 1000 + 5370 + 72
= **7267 calories.**

Example 2.—*The temperature of a pond is 8° C. when a freezing wind sets in. Assuming that the water parts with its heat at a uniform rate, compare the time taken to the beginning of freezing with the time taken to freeze the top half-inch of water, the total depth being 20 inches.* (*The latent heat of fusion of ice is 80 calories.*)

Solution.—We are only concerned with comparisons, and therefore we may confine ourselves to unit area of the pond, *i.e.* 1 sq. cm.

∴ Volume of water = 1 × 20 × 2·54 = 50·8 c.c.
∴ Weight of water = 50·8 grams.
Temp. of water = 8° C.
∴ Heat to be extracted to cool to freezing-point
= mass × specific heat × fall in temperature
= 50·8 × 1 × 8
= 406·4 calories.

Volume to be converted to ice $= 1 \times \frac{1}{2} \times 2 \cdot 54 = 1 \cdot 27$ c.c.

$\therefore$ Weight to be converted to ice $= 1 \cdot 27$ grams.

$\therefore$ Additional heat to be extracted to freeze this water

$= 1 \cdot 27 \times 80.$

$= 101 \cdot 6$ calories.

$\therefore$ Total heat now supplied $= 406 \cdot 4 + 101 \cdot 6 = 508.$

$\therefore$ Ratio required $= 406 \cdot 4 : 508$

$= 4 : 5.$

Latent Heat of Vaporisation by Method of Mixtures.—The theory of the "method of mixtures" for the determination of the latent heat of vaporisation of a liquid is very simple. The usual determination is that for water. Steam is produced in a flask A (Fig. 75), and is carried along a delivery tube into a copper condenser C immersed in a calorimeter B containing cold water. As the steam passes through, condensation takes place, and the condensed steam collects at the bottom of the condenser, the latent heat being given up to the water and the calorimeter. The rise in temperature is carefully noted.

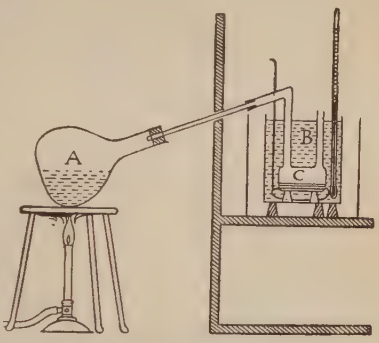

FIG. 75.—Apparatus for determining the Latent Heat of Steam.

In carrying out the experiment, the delivery tube from the boiler A is not immediately connected with the condenser; we wait until a steady flow of steam has been issuing for some time, and the connection, which is a loose one, so as to be made quickly, is then effected and the time noted. Steam now passes through the apparatus for a short time, and the tube from A is disconnected suddenly, the time being again noted. The temperature continues to rise a little after disconnection, owing to the fact that the water in the calorimeter does not absorb the heat from the condenser instantaneously, and the time at which the maximum temperature is reached is carefully noted.

By weighing the calorimeter, condenser, and contents before and after the experiment, the total amount of steam condensed can be found.

Let M = mass of steam condensed.
" L = its latent heat.
" t_2 = initial temperature of the steam (*i.e.* boiling-point corrected for barometric pressure).
" t_1 = initial temperature of calorimeter and contents.
" m = mass of water in calorimeter.
" c = its specific heat.
" m_1 = mass of calorimeter and condenser (both of same material).
" c_1 = specific heat of calorimeter and condenser.
" θ = final temperature of mixture (corrected for cooling losses).

Then heat lost by steam = heat gained by calorimeter and contents.

Now heat lost by steam = heat lost in changing M grams to steam at $t_2°$ (boiling-point) into water at same temperature + heat lost by M grams water at $t_2°$ in cooling to $\theta°$
$= ML + Mc(t_2 - \theta)$ calories.

Also heat gained by calorimeter and contents.

$$= (mc + m_1c_1)(\theta - t_1).$$

Hence we have

$$ML + Mc(t_2 - \theta) = (mc + m_1c_1)(\theta - t_1),$$

giving $$ML = (mc + m_1c_1)(\theta - t_1) - Mc(t_2 - \theta),$$

i.e., $$L = \frac{(mc + m_1c_1)(\theta - t_1) - Mc(t_2 - \theta)}{M} \text{ calories.}$$

Example 3.—6·32 *grams of steam were passed into* 102·8 *grams of water contained in a calorimeter whose water equivalent was* 4·5. *Find the latent heat of steam, given that the initial and final temperatures of the water in the calorimeter were* 13° *C. and* 47° *C. respectively.*

Solution.—Heat lost by hot steam = heat gained by cold water and calorimeter.

Let L = latent heat of steam.

Heat lost by steam = heat given up in condensing 6·32 grams of steam at 100° C. + heat given up by 6·32 grams of water at 100° C. in cooling to 47° C.

$$= 6{\cdot}32L + [6{\cdot}32 \times 1 \times (100 - 47)]$$
$$= 6{\cdot}32(L + 53) \text{ calories.}$$

Heat gained by cold water and calorimeter

$$= (102{\cdot}8 + 4{\cdot}5) \times 1 \times (47 - 13)$$
$$= 107{\cdot}3 \times 34.$$

Hence $6{\cdot}32(L + 53) = 107{\cdot}3 \times 34.$

$$\therefore \quad L + 53 = \frac{107{\cdot}3 \times 34}{6{\cdot}32}$$
$$= 577.$$
$$\therefore \quad L = 577 - 53$$
$$= 524 \text{ calories, a rather low result.}$$

Berthelot's Apparatus.—A practical difficulty occurs in the above experiment, due to the condensation of some vapour in the delivery tube before the steam reaches the condenser in the calorimeter. To avoid this, the tube is sloped **upwards** so that any condensed vapour may flow back to the boiler; but it is also advisable, as an additional precaution, to protect the delivery tube from its cold surroundings by swathing it in some non-conducting material.

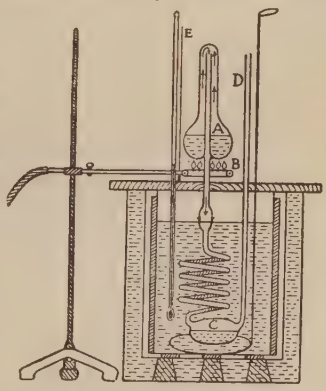

FIG. 76.—Berthelot's Apparatus.

In spite of these precautions, however, this is still a serious source of error, and with a view to overcoming this difficulty a modification of the apparatus has been devised by Berthelot.

The liquid of which the latent heat is required is boiled in a glass vessel A (Fig. 76) heated by means of a ring-burner B underneath it. The vapour is carried down a vertical tube, the upper

orifice of which is above the surface of the liquid. This tube is connected with a spiral tube immersed in the cold water in the calorimeter, and as the steam passes along this spiral tube it condenses, and is collected in a vessel C which communicates with the atmosphere by the tube D.

The calorimeter is suitably protected by jackets and lid from heat losses to the surroundings. These also protect the calorimeter from the direct heat of the ring-burner, whilst the thermometer is protected from the flame by means of a partition of asbestos E.

The calculation is very simple, and should offer no difficulty to the student.

Latent Heat of Ice by Method of Mixtures.—To determine the latent heat of ice by the method of mixtures, a suitably protected calorimeter of mass m_1 and specific heat c_1 is about half-filled with, say, m grams of water at a temperature t a little above the temperature of the room, say 20° C. A piece of ice weighing about 10 grams is then taken, quickly dried with a piece of blotting-paper, and introduced into the calorimeter. The whole is kept well stirred. As the ice melts, it extracts heat from the calorimeter and its contents, and the temperature therefore steadily falls. A small fraction of this fall is contributed by the heat losses of the apparatus to the surroundings, and this must be allowed for by a suitable correction.

Let θ be the final corrected temperature at which the mercury thread ceases to fall and begins to rise.

If M = mass of ice at 0° C.
and L = latent heat of ice,

then heat lost by the calorimeter and contents = heat gained by the ice.

Now the heat lost by the calorimeter and contents

$$= (mc + m_1c_1)(t - \theta) \text{ calories.}$$

Also the heat absorbed by the ice = heat necessary to convert M grams of ice at 0° into water at 0° + heat necessary to raise the temperature of M grams from 0° to θ°

$$= ML + Mc(\theta - 0) = ML + Mc\theta.$$

Hence $ML + Mc\theta = (mc + m_1c_1)(t - \theta)$.

$$\text{Whence } L = \frac{(mc + m_1c_1)(t - \theta) - Mc\theta}{M} \text{ calories.}$$

Example 4.—*A copper calorimeter weighing 22·5 grams (specific heat 0·095) contains 32·4 grams of water at 18°·2 C. A piece of ice is introduced weighing 5·8 grams. (The latent heat of ice is 80.) To what temperature will the water in the calorimeter fall?*

Solution.—Heat lost by calorimeter and water = heat gained by the ice.

Let t = final temperature required.

Heat lost by calorimeter and water

= (mass of water + water equivalent of calorimeter) × specific heat of water × fall in temperature

$= (32{\cdot}4 + 22{\cdot}5 \times 0{\cdot}095) \times 1 \times (18{\cdot}2 - t)$

$= 34{\cdot}54(18{\cdot}2 - t)$.

Heat gained by ice

= heat required to melt the ice + heat required to heat the melted ice from 0° to t° C.

$= (5{\cdot}8 \times 80) + 5{\cdot}8 \times 1 \times (t - 0)$

$= 464 + 5{\cdot}8t$.

$\therefore \quad 464 + 5{\cdot}8t = 34{\cdot}54(18{\cdot}2 - t)$.

$\therefore \quad t(5{\cdot}8 + 34{\cdot}54) = 34{\cdot}54 \times 18{\cdot}2 - 464$

$\therefore \quad 40{\cdot}34t = 164{\cdot}63$

$\therefore \quad t = \dfrac{164{\cdot}63}{40{\cdot}34} = 4°{\cdot}08$ C.

Application of Latent Heat Values to Specific Heat Determinations—Black's Ice Calorimeter.—The earliest and simplest, and probably one of the best means of determining the specific heat of a solid substance was due to Dr. Joseph Black, to whom we have already referred as the pioneer and founder of the study of calorimetry. He procured a block of ice with a perfectly flat upper surface, and worked a cavity into it (Fig. 77). He then took the substance whose specific heat he wished to find, heated it to a known temperature in a heater, and transferred it quickly into the cavity in the ice (previously dried). He now fitted a lid

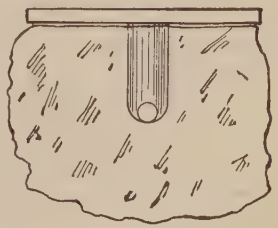

FIG. 77.

of ice firmly on to the top of the block so as to prevent outside heat from entering. He left this for some considerable time until the temperature of the body was reduced to 0° C. In this process of cooling, however, the heat was given up to the surrounding ice, some of which therefore melted into water at 0° C. This water was now poured out and carefully weighed. Suppose the mass of the hot body was 750 grams, and that its temperature was 100° C. Then the heat it gave up to the ice was $750 \times 100 \times$ its specific heat (call it c), *i.e.* $75{,}000c$ calories. Now suppose that 100 grams of ice were melted. Each gram required 80 calories, so that in all 8000 calories were required. Then $75{,}000c = 8000$, from which the specific heat of the solid works out to

$$\frac{75000}{8000} = 0{\cdot}094.$$

Bunsen's Ice Calorimeter.—A modern form of ice calorimeter is that due to Bunsen. It is illustrated in Fig. 78. The apparatus is essentially a sensitive form of dilatometer, into the bulb B of which is fused a large test-tube A. The stem leading from B is bent upwards as shown, and terminates in an enlargement D. This is fitted with a rubber stopper through which passes a graduated capillary tube DE bent so as to be horizontal. Imprisoned in the bulb B by a column of mercury CD is some cold water from which all traces of dissolved gases have been removed by boiling. The mercury thread completely fills the stem, and, by pushing in the stopper D sufficiently, it is made to reach about half-way along the capillary tube DE. It is necessary first to obtain a thick coating of ice round the bottom of the test-tube A. For this purpose the apparatus is immersed in an ice-bath F, and some ether is poured into A. By bubbling air through the ether rapid evaporation is induced, and the heat required for this is extracted from the water in B round A. A coating of ice is formed, and the presence of the ice-jacket F causes this to grow steadily until a sufficient thickness has been attained. Some ice-cold water is now introduced into A, and the solid whose specific heat is required is

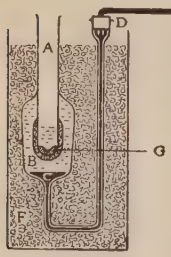

FIG. 78.—Bunsen's Ice Calorimeter.

introduced at a high temperature (from a heater). It is prevented from breaking the tube in falling by interposing at the bottom some protecting substance G, such as wool. A is then corked up to protect it from heat losses to the surroundings. The heat from the hot solid is gradually given up to the walls of the test-tube, and is thence transmitted to the crust of ice round it. Some of this therefore melts, but in so doing *contracts in volume*, thus forcing up the mercury level at C. In consequence the mercury thread in the capillary tube DE recedes. This will of course continue until the temperature of the solid in the test-tube has been reduced to 0° C., after which no more ice will melt on this account.

If t = the initial temperature of the hot solid,
if m = the mass of the solid,
if c = the specific heat of the solid,

then the heat given up to the calorimeter $= mc(t-0)$
$= mct$ calories.

Let L = the latent heat of fusion of ice.
„ v = the decrease in volume registered by the capillary thread during the experiment.
„ v_1 = the decrease in volume which occurs for every gram of ice melted.

Then $\frac{v}{v_1}$ = the mass of ice melted during the experiment.

∴ Heat gained by the ice $= \frac{v}{v_1}$L.

Hence we have $\frac{v}{v_1}\mathrm{L} = mct$,

giving $c = \frac{v}{v_1} \times \frac{\mathrm{L}}{mt}$ . . . (1)

The quantities on the right-hand side are all known except v_1. This must therefore be determined by a preliminary experiment. For this purpose we use a solid **whose specific heat we know,** so that using equation (1) v_1 is at once calculated.

Once this is determined, the value may be used for all subsequent experiments with the same instrument. The equation may be rewritten

$$c=\left(\frac{L}{v_1}\right)\frac{v}{mt} \quad . \quad . \quad . \quad . \quad (2)$$

and since L and v_1 are both constant, we have

$$c=K\frac{v}{mt}$$

where K is the constant of the instrument.

It is better, however, to avoid the actual use of the value for L, owing to the uncertainty which exists as to its exact value.

In the preliminary experiment, if M is the mass of the solid, C its known specific heat, and θ its initial temperature, then, denoting by V the observed decrease in volume during the experiment,

$$C=\left(\frac{L}{v_1}\times\frac{V}{M\theta}\right)$$

$$\text{giving } \frac{L}{v_1}=\frac{CM\theta}{V}$$

and equation (2) now becomes $c=\left[\frac{v}{mt}\times\frac{CM}{V}\right]$

We are thus independent of any actual values for L.

Although this apparatus is known as Bunsen's ice calorimeter, it should be understood that we are not bound to use ice. Any liquid which can conveniently be frozen is suitable, and as a matter of fact ice is *not* the most suitable. The temperature of the laboratory being many degrees above zero, there will always be a flow of heat from the surroundings to the instrument, so that even before the solid is introduced into the test-tube some of the ice-crust is constantly being melted. This means that there is a persistent and unavoidable movement of the mercury thread in the capillary tube when such movement is not wanted, and corrections have therefore to be applied on this account.

We may avoid the necessity for such corrections if it is possible to use a liquid whose freezing-point coincides roughly with the normal laboratory temperature of about 15° C. Acetic acid is such a liquid; its melting-point is 16° C., and it is therefore preferable to water.

Example 5.—*In experimenting with a Bunsen ice calorimeter, it was found that the mercury receded* 0·084 *cm. along the tube for every calorie of heat supplied.* 128 *grams of a substance at* 100° *C. was then transferred to the calorimeter, and a total movement of* 11·83 *cms. was recorded in the mercury column. What was the specific heat of the solid?*

Solution.—Let C = specific heat of solid.

Heat lost by hot solid on cooling from 100° C. to 0° C.
= mass × specific heat × fall in temperature
= 128 × C × 100
= 12,800C calories.

Movement of thread of mercury per calorie = 0·084 cm.
Total movement of thread of mercury per calorie = 11·83 cms.

$$\therefore \text{ Number of calories absorbed by ice} = \frac{11{\cdot}83}{0{\cdot}084} = 140{\cdot}8 \text{ calories.}$$

$$\therefore \ 12800C = 140{\cdot}8$$

$$\therefore \ C = \frac{140{\cdot}8}{12800} = 0{\cdot}011.$$

Joly's Steam Calorimeter.—Just as Bunsen utilised the latent heat of ice in his calorimeter, so Joly has made use of the heat given up by steam in condensing round a cold body in order to determine the specific heat of the body. His apparatus is shown in Fig. 79. Suspended from the left arm of a balance by means of a fine platinum wire is a copper or platinum pan B upon which the solid C whose specific heat is required is placed. Surrounding this is a steam chamber A suitably jacketed with a non-conducting material as shown. The initial

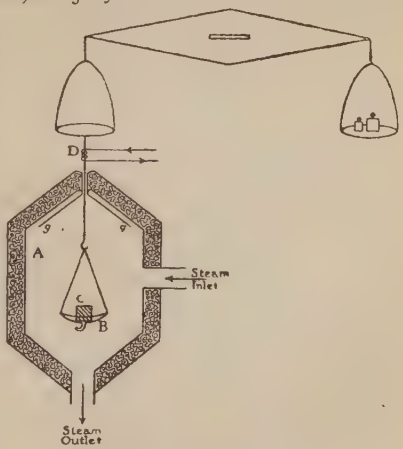

FIG. 79.—Joly's Steam Calorimeter.

temperature of the chamber having been noted, we balance the body by weights on the other pan.

Steam is now passed through the vessel, and as it comes into contact with the cold parts some of it condenses. The drops which form on the roof of the vessel are prevented from falling on the pan by the thin shield *gg*. Hence the drops which collect on the pan B are due solely to the pan and to the solid C. This condensation will continue until the temperature of the chamber has risen to that of the steam, say $t°$ C. (usually 100°).

We determine the mass of steam condensed by adding weights to the right-hand pan to produce balance.

Part of this weight is due to the solid, and part to the pan.

Let M = mass of steam condensed by the solid.
„ M_1 = mass of steam condensed by the pan.
„ L = latent heat of vaporisation of water.
∴ Heat given up by the steam = $(M + M_1)L$ calories.

Also, if m = mass of solid,
if c = its specific heat,
if m_1 = mass of pan,
if c_1 = its specific heat,

then heat gained by solid and pan = $(mc + m_1c_1)(t - t_1)$ calories.

Hence we have $(mc + m_1c_1)(t - t_1) = (M + M_1)L$, from which c may be calculated if c_1 be known. This entails a preliminary experiment in which the pan alone is used, the solid C being removed.

We now have $m_1c_1(t - t_1) = M_1L$, from which c_1 can at once be determined.

The advantage of this method is that the whole experiment consists of weighings. It is therefore capable of great accuracy.

In practice it is found that the suspending wire causes a little additional condensation, giving rise to serious errors. To avoid this a small platinum coil D is wound round it as shown, and a current is passed just sufficient to maintain the wire at 100° C., thus preventing condensation from taking place.

Example 6.—*A copper ball* 84·4 *grams in weight and at a temperature of* 4° *C. is exposed to a stream of dry steam at* 100° *C. in Joly's calorimeter. As a result,* 1·42 *grams of steam are found*

to condense on it. Given that the latent heat of steam is 536 *calories, find the specific heat of the copper.*

Solution.—Let C = specific heat of copper.

The heat gained by the copper ball in heating from 4° C. to 100° C.

= mass of copper × its specific heat × rise in temperature
$= 84{\cdot}4 \times C \times (100 - 4)$
$= 84{\cdot}4 \times 96 \times C$ calories.

This is extracted from the steam to condense 1·42 grams of it.

$$\therefore \quad 84{\cdot}4 \times 96 \times C = 1{\cdot}42 \times 536.$$

Heat extracted from steam $= 1{\cdot}42 \times 536$.

$$\therefore \quad C = \frac{1{\cdot}42 \times 536}{84{\cdot}4 \times 96} = \mathbf{0{\cdot}094}.$$

Differential Form.—In the experiment as above described, it will be noticed that *two* experiments are necessary: one for the pan alone, and one for the pan plus solid. To avoid this, Joly introduced an important modification known as the **differential calorimeter. Two** pans are used alike in weight and material, the one containing the solid, and the other being empty (Fig. 80). The steam chamber now surrounds both pans, and the weights which must be added to the top right-hand pan for balance after steam has passed through the apparatus at once gives the amount of steam condensed by the solid alone. As a matter of fact, however, it is practically impossible to obtain the two pans exactly alike; and we therefore repeat the experiment with the solid in the second pan instead of the first. The errors in the two cases being equal and opposite, they neutralise each other.

This is the most accurate method known for the determination of the specific heat of a solid. It should be noted that the vapour of any suitable liquid may be substituted for the steam.

Specific Heat of a Gas at Constant Volume by Joly's Differential Calorimeter.—One of the most useful applications to which Joly's differential calorimeter has been put has been to determine the specific heat of a gas at constant volume. It will be remembered that a method of mixtures was described for this, but the method is unsatisfactory. The modification of Joly's differential

calorimeter necessary for the determination of c_v is simple. Two copper spheres, as far as possible identical in size and mass, are suspended over the pans. One sphere contains the gas under test at about twenty atmospheres, and the other is evacuated.

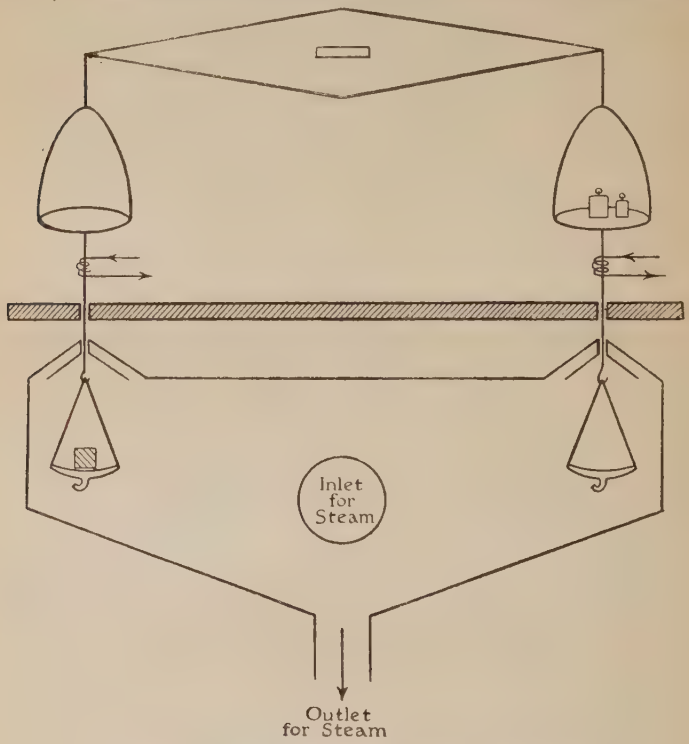

FIG. 80.—Joly's Differential Steam Calorimeter.

The experiment is carried out exactly as before, two values being obtained: one with the gas in the left-hand sphere, and the other with the gas in the right-hand sphere. The mean gives the required result.

Liquids.—If instead of gas we fill one sphere with liquid, we are at once able to determine its specific heat by this method.

Example 7.—*In an experiment with Joly's differential steam calorimeter the contents of one of the spheres of copper were* 2·1427 *grams of air at* 15°·5 *C. The other sphere was evacuated. The current of steam raised the temperature of the spheres to* 100° *C., when balance was restored to the arrangement by the addition of* 0·058 *gram. Find the specific heat of air, given that the latent heat of steam is* 536.

Solution.—The addition of the weight of 0·058 gram gives the amount of steam condensed by the air alone.

$\therefore$ Heat extracted from the steam to heat the air in the sphere

$$= 0{\cdot}058 \times 536.$$

And heat gained by the air = mass of air in sphere × specific heat of air × rise in temperature

$$= 2{\cdot}1427 \times c_v \times (100 - 15{\cdot}5),$$

where c_v = specific heat of air at constant volume.

$$\therefore \quad 2{\cdot}1427 \times 84{\cdot}5 \times c_v = 0{\cdot}058 \times 536.$$

$$\therefore \quad c_v = \frac{0{\cdot}058 \times 536}{2{\cdot}1427 \times 84{\cdot}5}$$

$$= 0{\cdot}172.$$

EXAMPLES ON CHAPTER XI

1. What is meant by the statement that the latent heat of water is 80 ? The temperature of a pond is 8° C. when a freezing wind sets in. Describe what happens as the water parts with its heat.
2. Define the terms "latent heat of vaporisation" and "latent heat of fusion" for any given substance. How much heat is required to raise the temperature of 15 grams of fat from 15° C. to 70° C., given melting-point = 45° C., latent heat = 25, and specific heat in both solid and liquid states = 0·5 ?
3. 1 lb. of hot water is poured into a shallow polished vessel supported on three corks, and whilst 0·25 oz. is evaporating the temperature falls from 90° C. to 80° C. Neglecting heat losses to the surroundings, calculate the latent heat of vaporisation of water.

4. By how much will the weight of a kilogram of iron, whose initial temperature is 15° C. and whose specific heat is 0·12, increase when weighed in steam at 100° C. ?
5. Steam at 100° is passed into a copper calorimeter weighing 100 grams and containing 500 grams of water at 15°, until the temperature of the calorimeter and its contents rises to 25°. Calculate the weight of steam condensed, given the specific heat of copper = 0·1 and latent heat of steam = 536.
6. 100 grams of iron at 50° C. are placed in a vessel containing 1000 grams of water at 0° ; how many grams of ice at 0° must be added to reduce the temperature of the mixture to 0° ? All the ice is supposed to be melted. (Specific heat of iron = 0·113 ; latent heat of fusion of ice = 80.)
7. If the specific gravity of ice is 0·918, at what rate per square metre is heat escaping from a lake when a layer of ice 2 mm. thick is formed in an hour on its surface ?
8. Compare the quantities of water, originally at 15° C., necessary to condense 100 tons of steam at 39° C. when its total latent heat is 580, and at 26° C. when its total latent heat is 588.
9. The boiling-point of a liquid is 156°, its mean specific heat is 0·46, and its latent heat is 68. Find the quantity of vapour at the boiling-point that must be passed into a copper vessel (specific heat 0·1) weighing 30 grams, which contains 250 grams of the liquid at 15°, in order to raise the temperature of the latter to 27°.
10. How may the knowledge of latent heat value be utilised to determine the specific heat of a solid ? Give full experimental details.
11. What is meant by the latent heat of steam ? Explain some method by which its value can be found.
12. "The latent heat of steam is 536." Explain the meaning of this statement, defining any scientific terms you may use. If heat be introduced into ice-cold water contained in an open flask with non-conducting walls at a uniform rate such as at first to raise its temperature by 1° per minute, how long would it take to convert the water completely into steam ?
13. What do you understand by capacity for heat, latent heat, specific heat ? A copper calorimeter weighing 100 grams contains 400 grams of water at 18° C. Steam at 100° C. is passed into the water, and the temperature rises to 34° C. The weight of the calorimeter and water after the experiment is found to be 511 grams. Calculate the latent heat of steam, the specific heat of copper being 0·1.
14. 8 grams of ice at 0° are dropped into 64 grams of paraffin oil at 20°, and are just melted in cooling the oil to 0°. What

is the specific heat of the oil ? (*N.B.*—The latent heat of fusion of ice is 80.)

15. If the latent heat of fusion of ice is 80, and its density at 0° is 0·917, find the travel of the mercury in the tube of a Bunsen ice calorimeter when 10 calories are given to the ice, the diameter of the tube being 0·4 mm.

16. 1 gram of metal heated to 100° is dropped into a Bunsen ice calorimeter in which the weight of mercury required to fill 1 cm. of the index tube has been found to be 0·026 gram. The thread of mercury moves through 52·5 mm. What is the mean specific heat of the metal ? (1 gram of water in freezing expands 0·0907 c.c., and its latent heat of fusion is 80·02. The density of mercury is 13·6.)

17. Describe an accurate method for determining the specific heat of a gas at constant volume.

18. A calorimeter of iron weighs 149 grams empty, and 401 grams when it contains a quantity of water at 9° C. Steam is passed into the water until the temperature rises to 63° C., when the calorimeter is found to weigh 426 grams. If the specific heat of iron is 0·113, calculate the latent heat of vaporisation of water.

19. The melting-point of tin is 232° C. Its latent heat of fusion is 14 calories, and its specific heat is 0·055. How many units of heat will melt 100 grams of tin originally at 20° C. ?

20. In a central heating apparatus steam is used. Given that the latent heat of steam at 100° C. is 537 calories, find how much heat is given up by the steam per minute if 4 kilograms of steam per minute at 100° C. is condensed into water at 65° C. in its circuit through the building.

21. Assuming that the latent heat of vaporisation of petrol is 160 B.T.U.'s, calculate the heat absorbed by the vaporisation of 1 gallon. (The specific gravity of petrol is 0·7, and 1 gallon of water weighs 10 lb.)

22. What weight of steam at 100° C. will be required to be passed into a mixture of 100 grams of ice and 100 grams of water at 0° C. in order to obtain a final temperature of 10° C. ? (Latent heat of ice = 80, and of steam = 537.)

23. Assuming that the specific heats of paraffin wax in the solid and the liquid states are known, how would you determine experimentally the latent heat of fusion ?

24. Describe an experiment for the determination of the latent heat of vaporisation of water, and state your precautions against error. Illustrate, by planning out a numerical example, how you would deduce the result by calculation.

25. 51·2 grams of water at 15° C. are placed in a beaker, and the whole is heated at a uniform rate. After 3 minutes 55 seconds boiling begins, and after another 26 minutes

15 seconds the last trace of water is boiled away. Deduce a value for the latent heat of vaporisation of water.

26. A body weighing 47·5 grams is heated to 97° C. and then introduced into a Bunsen's ice calorimeter. The mercury thread indicates a contraction of 0·51 c.c. If the density of ice is 0·92, and the latent heat of fusion of ice is 80, find the specific heat of the material of the body.

27. 15 grams of phosphorus are carefully melted and cooled to 28° C., when solidification sets in. How much will remain molten, given that the normal melting-point is 44° C., the latent heat of fusion 5·4, and the specific heat of molten phosphorus is 0·2 ?

28. In an experiment with a Bunsen ice calorimeter the decrease in volume of ice and water at 0° C. is measured by the movement of the mercury in a capillary tube of 1 mm. bore. When a piece of metal 2 grams in weight is heated to 100° C. and dropped into the calorimeter, the mercury is found to recede a distance of 2·93 cms. along the tube. Given that $\pi = 3{\cdot}14$, the density of ice $= 0{\cdot}916$, and the latent heat of ice $= 79{\cdot}25$, find the specific heat of the solid.

29. Calculate the latent heat of fusion and of vaporisation in the British system of units if the corresponding values in the C.G.S. system are 80 and 537 calories respectively.

CHAPTER XII

MODES OF TRANSFERENCE OF HEAT—CONVECTION

The Three Modes of Transference of Heat.—Heat may be transferred from one point to another by any of three different ways. These are known respectively as :

1. Convection.
2. Conduction.
3. Radiation.

Convection.—In convection the heat is transferred as a result of the movement of comparatively large masses of a substance from a point of higher temperature to a point of lower temperature. We use the term " large masses " in the sense of their containing millions and millions of molecules. We have already considered one or two simple examples of convection currents (Chapter III.), and it will be remembered that owing to the difference in density set up by a rise in temperature locally, the hotter bodies of molecules are forced upwards whilst the colder bodies of molecules originally removed from the heat source take their place and become in their turn heated. Convection is therefore only possible in a substance whose molecules are sufficiently loose with respect to each other to enable them to move from one part of the substance to another, *i.e.* convection occurs in liquids and in gases, but not in solids.

Conduction.—Conduction differs from convection in that the heat is transmitted from one point in a substance to another without any actual movement of a molecule away from its mean position. We know from the Kinetic Theory that the molecules are in a state of agitation, and that the heat which a body possesses is the measure of the amount of this molecular agitation : the temperature increases when the kinetic energy of translation of the molecules increases. When one end of a poker is placed in

the fire, its temperature goes up, *i.e.* the molecules at that end vibrate more rapidly. By repeated collisions this increased vibration is transmitted along the poker from molecule to molecule, causing a gradual increase in temperature along its length. The heat is said to be *conducted* along the rod.

Radiation.—In radiation the heat is transmitted from one point to another which is entirely separated from it, often by a great distance. Thus heat is transmitted from the sun to the earth, a distance of about 93 millions of miles, by radiation. Similarly, the heat we receive when standing in front of a fire or any other heat-source is due to radiation. At the one point the heat is *emitted*, and at the other it is *absorbed*. The agency whereby the phenomenon of radiation is rendered possible is the *ether*, the medium which pervades all space; and the means by which the radiation is propagated through the ether is *wave motion*. The electrons within the molecules of a body which is heated are in a state of vibration, and these vibrations react on the ether in contact with the molecules, setting up waves. When a body is interposed in the path of a heat wave, *i.e.* when the ether vibrations impinge on the interposing body, the electrons within the molecules of the body themselves begin to vibrate "in tune" with the ether and the body emitting the heat, and this molecular vibration manifests itself as a rise in temperature—we say the body is *absorbing* the heat radiations.

We shall deal fully with the phenomena of conduction and radiation in succeeding chapters. In this chapter we shall confine ourselves to a study of convection and its effects.

Examples of Convection in Liquids—The Boiling of Water.—When a liquid is boiled in a vessel, the heat is caused to be uniformly distributed throughout the body of the liquid through the agency of convection currents. This has been fully dealt with in Chapter III., and further details are unnecessary. It is interesting to note, however, what would be the consequence of applying a flame near the top of a volume of liquid in a flask, instead of to the bottom.

Since the currents of heated liquid are upwards, the heat from the flame is circulated over the very small range of the liquid above the level of the flame. Being much hotter, and therefore lighter than the lower layers, it remains on top, and the heat can

only penetrate downwards by conduction. Water is a bad conductor of heat, and consequently it would take a very long time indeed for the whole of the liquid to boil in the case of water: while the top is boiling, the bottom will remain quite cold for a long time. Hence when general heating is desired, the flame should always be applied under the vessel.

Water-Circulation by Convection.—Suppose we have an arrangement as shown in Fig. 81, consisting of a corked flask A

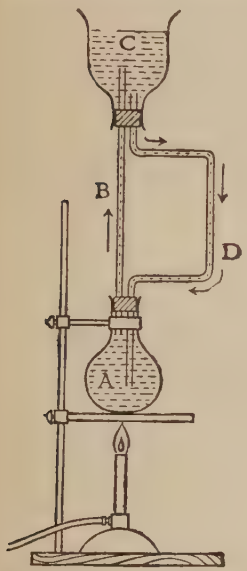

Fig. 81.—Hot-Water Circulation by Convection.

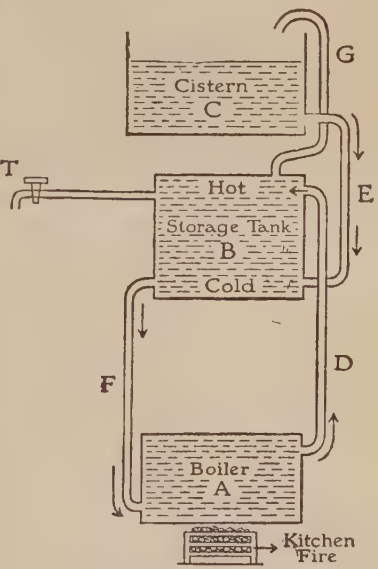

Fig. 82.—Hot-Water Supply in Houses.

filled with water, an inverted bell-jar C filled with cold water tinted with some colouring matter, and two connecting tubes B and D, one bent as shown. If we heat the water in A by means of a Bunsen burner its density will become less than that of the cold water in C and D, and a convection current will be set up. The hot water therefore rises up the tube B, and a cold current descends from C down D to take the place of the hot. This in

its turn becomes heated, and rises up B to give place to some more cold water from C. The motion of the current is indicated by watching the movement of the tinted water along the tubes.

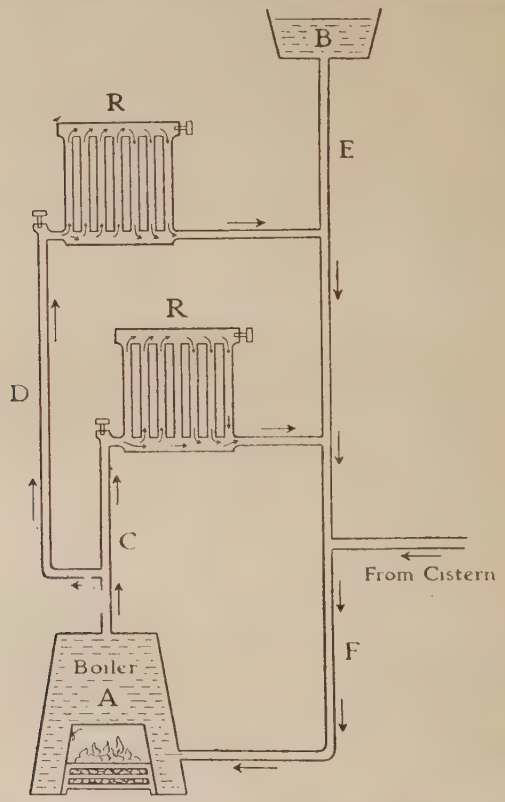

Fig. 83.—Heating a Building by Hot-Water Circulation.

Hot-Water Supply in Houses.—The above principle has very important domestic applications, some of which may now be considered. A slight rearrangement of Fig. 81 shows the means by which hot-water taps are worked in private houses. For the flask A we substitute a boiler A (Fig. 82), and instead of the bell-

jar we have a cistern C; for the Bunsen burner we substitute a kitchen fire. Additional to these we have a storage tank B from which the hot water is drawn off as required by means of the tap T. The arrangement works as follows:

Owing to the kitchen fire, differences of temperature between the cistern and the boiler A are set up, and a convection current is produced. Hot water rises from A up D into the upper part of the storage tank B, whilst the cold water takes its place from the lower part of B down F. Being lighter than the cold water in B, the hot water remains in the upper part of the tank. As it is drawn off at T more cold water automatically replaces it from C down E, and so the circulation is maintained. G is a safety pipe which returns to C any water violently thrown up during the boiling in B.

The Heating of Buildings by Hot Water.—Large buildings are very frequently heated by a hot-water circulation, the principle being identical with that described in the two previous articles. A boiler A (Fig. 83) situated somewhere in the basement of the building creates the necessary difference of temperature between boiler and cistern, and a convection current is set up, hot water moving upwards along pipes C and D, and cold water returning down the pipe EF. Periodically along the pipes, in each room, radiators R are placed, consisting of a number of short vertical pipes grouped so as to increase the total heating surface. Any of these may be cut off from, or included in, the general circulation by means of regulating valves. Cool air is usually brought into the rooms from outside through the walls along ducts which emerge behind the radiators, and in passing through the spaces between the vertical pipes of the radiator they get warmed. A supply of warm fresh air is thus assured.

Examples of Convection in the Atmosphere—Chimney Draught over a Fire—Ventilation.—A simple domestic example of convection currents in the air is afforded by the chimney draught over a fire (Fig. 84). The effect of the fire is to raise the average temperature of the column of air and smoke in the chimney above that of the air outside. Hence a current of air is drawn upwards whose volume is proportional to the excess of temperature. Hence the higher we raise the temperature of the chimney, the fiercer will be the draught. The larger part of

this air current will of course pass upwards without entering the fire, owing to the large opening above it. As, however, the oxygen of the air is required on lighting the fire, it is usual to force the air to pass through the coals by covering up the opening with a screen (shown dotted in Fig. 84). As soon as the fire is well alight, the screen is removed.

The chimney draught is very important as being the means whereby a room is adequately ventilated; fresh air being drawn in through the doors and windows, and foul air being carried off up the chimney. When, however, no special provision is made, this simple form of ventilation gives rise to objectionable cold

FIG. 84.—Chimney Draught.

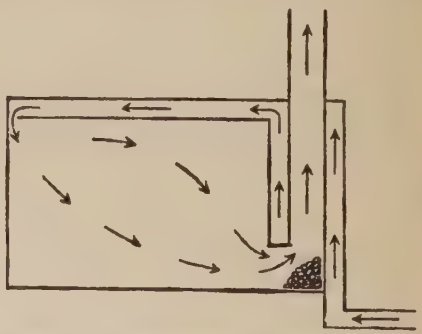

FIG. 85.—Ventilation of Room by Inlet Pipe round Chimney.

draughts. The remedy for this is to provide some simple means of guiding the incoming cold fresh air in such a way as to ensure a maximum of circulation with a minimum of draughts. In the Tobin system a ventilation pipe directs the air upwards towards the ceiling, where it spreads and becomes warmed, and then descends. Another method of warming the incoming fresh air is to guide it up a pipe surrounding the hot chimney, and to discharge it into the room from that part of the ceiling which is opposite to the chimney. This is illustrated in Fig. 85.

Winds—Land- and Sea-Breezes.—All atmospheric movements are consequences of convection currents, and one of the simplest cases of a periodic kind is that which is experienced daily at the

seaside, in which a sea-breeze blows by day, and a land-breeze by night (Fig. 86).

During the daytime the sun's rays fall on land and sea, but, owing to the high specific heat of water and the comparatively

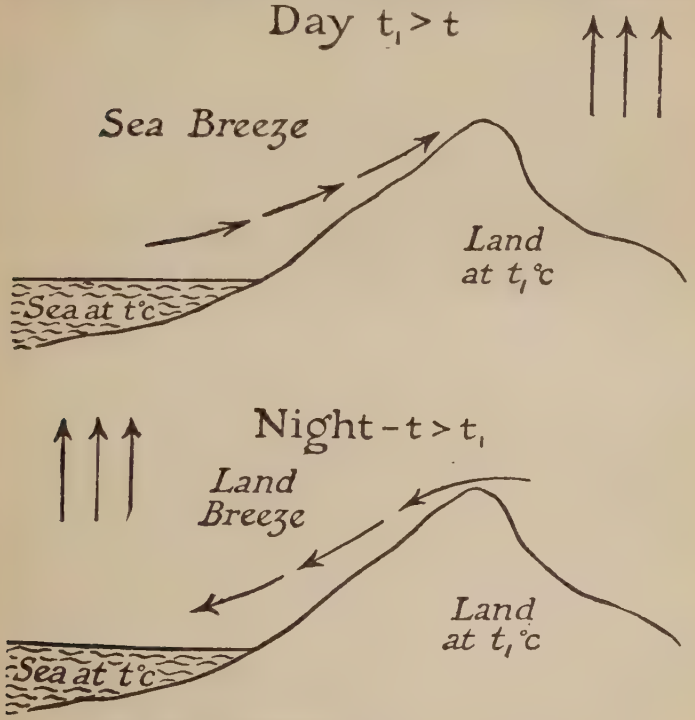

FIG. 86.—Production of Land- and Sea-Breezes.

low specific heat of rock, the temperature of the land quickly becomes higher than that of the sea. This heat is communicated to the air above the land, which is therefore at a higher temperature t_1 than that of the air above the sea. The hotter air, being less dense, rises, and its place is taken by the colder air from the sea. Accordingly we get the bracing sea-breeze which constitutes one of the attractions of seaside resorts.

At night the sun has gone down, and cooling takes place. The sea radiates its heat very slowly, whilst the land radiates its heat rapidly. Accordingly the air over the land quickly becomes cooler than that over the sea. The sea air therefore ascends, and causes a current of cool air to take its place from the land.

Land- and sea-breezes are not always well defined, owing to complicating factors which often arise. They are best felt along coasts which run north and south, and which are backed by mountains as in South America and West Africa. Certain strips of coast along parts of West Africa are infested with malaria germs (less so now than formerly), and in such regions the land- and sea-breezes are very important. At night no ships should be within ten miles of the coast, to avoid the malaria-laden land-breeze. Only during the daytime is it wise for them to venture into port.

Periodic Winds.—When a portion of the earth's surface is heated by the sun, the air becomes lighter and rises, its place being taken by the cooler and denser surrounding air. Generally speaking, the coldest regions of the earth are the poles, and the warmest the equator. Hence the general effect should be that cold winds as a whole will blow from the poles towards the equator, whilst in the upper regions of the atmosphere we should expect to find compensating return winds blowing from the equator to the poles.

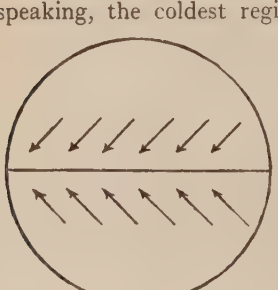

Fig. 87.—Trade Winds.

Owing, however, to the rotation of the earth on its axis, this direction due north and south does not obtain. As we get farther away from the poles, the various points on the earth are carried eastward with greater and greater speeds, causing the wind to veer more and more to the east. Hence, instead of winds blowing due N. and S., we have them in the regions of latitude 30° N. and S. blowing from the N.E. and S.E. respectively (Fig. 87). With one qualifying factor, this is a steady state of affairs, and the **trade winds,** as they are called, blow steadily all the year round,

As they approach the equator the temperature increases, and they therefore rise and form a return current in the upper atmosphere. Hence, between the trade winds there exists a belt of calms called the **doldrums.**

The qualifying factor above referred to is the fact that the sun is not vertically above the equator all the year round, owing to the inclination of the plane of the ecliptic (or the sun's apparent path in space) to the plane of the equator. In consequence of this the permanent wind systems of the earth move north and south with the seasons, the motion being about 15° on the average.

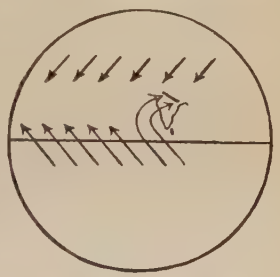

Fig. 88.—South-West Monsoons of India.

Monsoons.—Monsoons are periodic winds caused initially by the "swing-swang" of the trade winds north and south with the seasons. Just as land- and sea-breezes are created by night and day alternations in temperature, so monsoons are caused by summer and winter alternations in temperature. In the summer the sun has moved northwards, so that the S.E. trade winds cross north of the equator (Fig. 88). During the summer months, however, the land masses of India as a whole are being heated above the temperature of the sea, so that the air above the land becomes lighter and creates a centre of low pressure. In consequence of this, the air rises, and in order to take its place the S.E. trade winds are deflected as shown in the figure, and become a S.W. **monsoon.** Being laden with moisture derived from the sea atmospheres, they are a very welcome annual visitor. A reverse monsoon is, of course, created when the N.E. trade-wind system moves southwards with the sun in the winter.

Monsoons occur wherever land masses are interposed in the path of a periodic wind.

There are many other periodic winds occurring in different parts of the earth, all arising as convection currents in consequence of definite distributions of pressure and temperature belts, but those given should suffice to illustrate the important influence of convection phenomena upon climate.

EXAMPLES ON CHAPTER XII

1. Explain how heat is conveyed through a liquid by convection, and give some practical applications of this phenomenon.
2. Is it possible to have water in a test-tube such that the portion at the surface is boiling, whilst that at the bottom is frozen ?
3. Explain the differences between the distribution of heat by conduction, convection, and radiation. How would you arrange to show most clearly the occurrence of convection (*a*) with water at about 20° C., and (*b*) with water at about 0° C. ?
4. How is heat transmitted by radiation ? Describe some simple experiments to show the difference between the transmission of heat by radiation and by convection.
5. How is the principle of convection utilised in the heating of buildings by hot water ? Discuss the relative advantages and disadvantages of this method as compared with heating by open fires.
6. What are the general considerations which should guide you in designing a suitable means of ventilation of a house ? What influence has the " open fire " system on this ?
7. What is meant by convection ? Explain the effect of convection in producing regular atmospheric currents such as trade winds.
8. Explain the cause of land- and sea-breezes. What are the best natural conditions for their production, and under what circumstances may they become important ?
9. Explain how a stove in one part of the room heats the air in all parts of the room.

CHAPTER XIII

THE TRANSFERENCE OF HEAT—CONDUCTION

Transference by Conduction.—It was pointed out in Chapter XII. that heat is said to be transferred from one point in a substance to another by conduction when no motion takes place of groups of molecules as a whole. In this respect lies the chief difference between conduction and convection. In convection large groups of molecules move bodily from one part of the substance to another, carrying the heat energy with them ; in conduction each molecule is more or less " chained " to some mean position, but transmits some of its molecular agitation, *i.e.* its heat energy, to its neighbour by collision. This is passed along the line of molecules, so that if one end of a bar be heated, the other end will in time experience a rise in temperature. Initially, the specific heat of the substance is an important factor, since when the heat is supplied at one end, a body with a low specific heat will increase in temperature at a greater rate than one whose specific heat is high, whatever the relative powers of conducting the heat may be. The heat which is initially supplied performs several functions. Part of it is absorbed by the body, by virtue of its specific heat, to raise its temperature ; part of it is transmitted along the body by conduction ; and part is radiated or emitted from the sides of the body to its surroundings.

In consequence of the first of these factors, we find that different points along the body increase in temperature. After a time, however, this increase ceases, and a *steady state* is attained, the body being hot at one end and cooler and cooler as we get to the other end. The heat capacity as a factor need no longer be considered, the heat being now conducted along the body, or emitted from its sides, as fast as it is received. The proportion which is emitted to that which is conducted along the body is

very small, and accordingly when the steady state has been reached, provided suitable precautions are taken to diminish emission losses, we have a suitable means of investigating the laws underlying conductivity phenomena.

General Theory of Conduction.—Suppose we have a wall of material (Fig. 89), of thickness x, with parallel faces, and that on it we mark out a portion whose area is a. Let the two surfaces be at different temperatures, that of the front face $t_1°$ being at a higher temperature than that of the back face $t_2°$. Then the molecules nearer the front face will have more kinetic energy than those nearer the back, so that the tendency will be for this energy to be transmitted by conduction from the hotter to the cooler face.

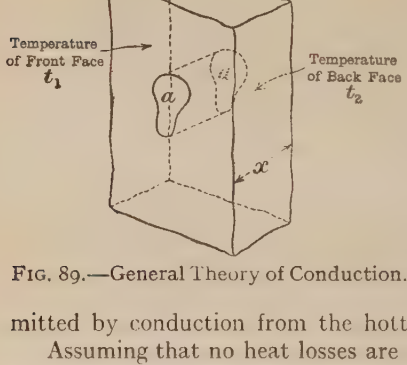

FIG. 89.—General Theory of Conduction.

Assuming that no heat losses are incurred by radiation to the surroundings, the amount of heat which will flow from front to back will be governed by the following factors :

1. *Time.*—Obviously the longer is the period during which heat is transmitted through the slab, the greater will be the quantity of heat flowing through.

2. *Area of Slab.*—The quantity of heat will also be proportional to the area of the slab a, for if we double the area we double the number of molecules which take part in the transmission of heat, and so on.

3. *Excess of Temperature between the two Faces.*—We have seen that the ability to transmit heat energy across the slab is consequent upon the fact that one face is at a higher temperature than the other. Hence the greater is this excess of temperature between the two faces, the greater will be the quantity of heat conducted through in any given time.

4. *The Thickness of the Slab.*—The influence of the thickness x of the slab is different from that of the factors we have already considered; for if we increase x, then the heat has to be transmitted

through a greater distance, and the quantity of heat which flows through from one face to the other *in the same time* will be less than before.

Hence the quantity of heat conducted through will be *inversely proportional* to the thickness.

∴ If H denotes the quantity of heat transmitted from one face to the other in a time T seconds, we have

$$(1)\ H \propto T,$$
$$(2)\ H \propto a,$$
$$(3)\ H \propto t_1 - t_2,$$
$$\text{and } (4)\ H \propto \frac{1}{x}$$

Combining these, we have

$$H \propto a\left(\frac{t_1 - t_2}{x}\right)T,$$

$$\text{or, } \mathbf{H = Ka\left(\frac{t_1 - t_2}{x}\right)T,}$$

where K is some constant.

If $t_1 - t_2 = 1°$, $a = 1$ sq. cm., $x = 1$ cm. and $T = 1$ second, then we have $H = K$.

Hence **the constant K may be defined as the amount of heat which is transmitted per second across a slab of material whose thickness is 1 cm., whose area of cross-section is 1 sq. cm., and whose difference of temperature between the two faces is 1° C.**

This constant is called the thermal conductivity of the material. —It is found that K varies in value from material to material, *i.e.* some bodies conduct heat much more readily than others, and we therefore speak of good conductors and bad conductors. Most of the metals are good conductors, whilst glass, wood, paper, etc., are bad conductors. In the formula

$$H = Ka\left(\frac{t_1 - t_2}{x}\right)T$$

we speak of $\frac{t_1 - t_2}{x}$ as the temperature gradient; it is the fall in temperature per unit length of the body. We therefore have:

Heat transmitted per second
$= K \times$ *area of cross-section* $\times$ *temperature-gradient.*

Example 1.—*A tank, 600 cms. long and 400 cms. wide, is filled with water which is covered with a layer of ice 6·6 cms. thick. If the temperature of the air is −10° C., and the thermal conductivity of ice is 0·0023, how much heat will be transmitted through the ice to the air per hour?*

Solution.—Here conduction takes place across the ice-slab.

Temperature of "hot" face $= 0°$ C.
Temperature of "cold" face $= -10°$ C.
Thickness of slab $= 6·6$ cms.

$\therefore$ Temperature gradient $= \frac{0-(-10)}{6·6} = \frac{10}{6·6}$

$\therefore$ Heat transmitted per second = conductivity coefficient × area of slab × temperature gradient

$$= 0·0023 \times 600 \times 400 \times \frac{10}{6·6}$$

$\therefore$ Heat transmitted per hour

$$= 0·0023 \times 600 \times 400 \times \frac{10}{6·6} \times 3600$$

$$= \mathbf{3,010,000 \text{ calories.}}$$

Example 2.—*A man is wearing a single garment whose fabric is 0·3 cm. thick, and whose conductivity coefficient is 0·000122. If the temperature of the air is 5° C., and the man's body temperature is 30° C., how much heat is lost to the surroundings per sq. cm. of surface per minute?*

Solution.—Temperature gradient $= \frac{30-5}{0·3} = \frac{25}{0·3}$

$\therefore$ Heat transmitted per second = K × A × temperature gradient

$$= 0·000122 \times 1 \times \frac{25}{0·3}$$

$\therefore$ Heat transmitted per minute $= 60 \times 0·000122 \times \frac{25}{0·3}$

$$= \mathbf{0·61 \text{ calories.}}$$

Example 3.—*Steam at 227° F. is conveyed along bare piping of 3 inches diameter and $\frac{1}{8}$ inch thickness. The inside temperature is 50° F., and the external conductivity of the piping is 0·0051 B.T.U.'s*

per square foot of outside surface per degree Fahrenheit excess of temperature. If the latent heat of steam is 950 *B.T.U.'s per lb., how much steam is condensed per* 100 *feet of piping per second?*

Solution.—The temperature gradient $= \frac{227 - 50}{\frac{1}{8 \times 12}} = 177 \times 96.$

Area of piping per 100 feet of length $= 100 \times 3{\cdot}14 \times \frac{3}{12} = 25 \times 3{\cdot}14$ and $K = 0{\cdot}0051$.

$\therefore$ Heat lost per second $= 0{\cdot}0051 \times 25 \times 3{\cdot}14 \times 177 \times 96$ B.T.U.'s.

Now 1 lb. of steam must lose 950 B.T.U.'s in order to condense.

$\therefore$ Steam condensed per hour $= \frac{0{\cdot}0051 \times 25 \times 3{\cdot}14 \times 177 \times 96 \text{ lb.}}{950}$

$= 7{\cdot}15$ **lb.**

Examples of Conduction.—We continually meet with the consequences of conductivity in our everyday experiences. Thus, in the winter, it is much more comfortable to handle wood than metal, although both are at the same temperature. This is because the body is at a higher temperature than either wood or metal, and, in touching them, heat tends to flow from the warmer hand to the cooler wood or metal, and since the metal is a good conductor of heat, and the wood is a poor conductor, the metal extracts the heat from the hand much more quickly than the wood. Conversely, in summer, when the sun has been shining on wood and metal, the metal, on being touched, transfers heat to the fingers more readily than does the wood, and it therefore feels hotter.

Still air is a bad conductor, and the main object of clothing is to imprison a layer of this air round the body, so as to prevent the warmth of the body from being lost to the surroundings. For this reason, woollen clothing, the texture of which enmeshes air, is more useful than cotton and calico clothing which do not. Metal teapots are provided with wooden handles, as otherwise it would be impossible to hold them with comfort. Wire-gauze is placed over a Bunsen flame, because it conducts the heat away too rapidly to allow the temperature above the gauze to reach the ignition-point for the gas. All are familiar with the fact that by applying a match over the gauze instead of under it, we get a flame on top, but not underneath. This is the principle of the miner's safety-lamp. By surrounding the flame with wire-gauze, the temperature

outside the gauze is kept below the ignition-point for the inflammable gases which are known to form in the atmosphere of the mines.

Comparison of K for Different Metals.—Two pieces of apparatus, both depending upon the same principle, may be mentioned for comparing the conductivities of different metals.

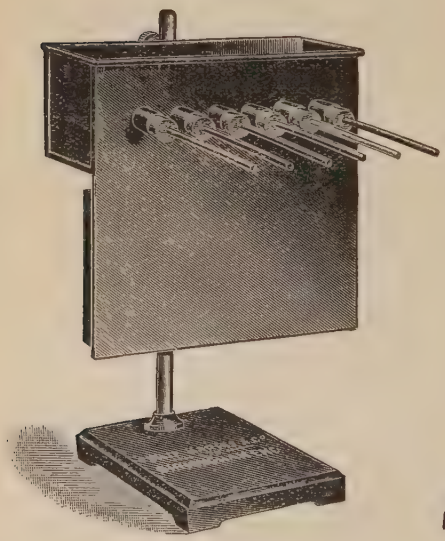

FIG. 90.—Irgenhouz's Apparatus.

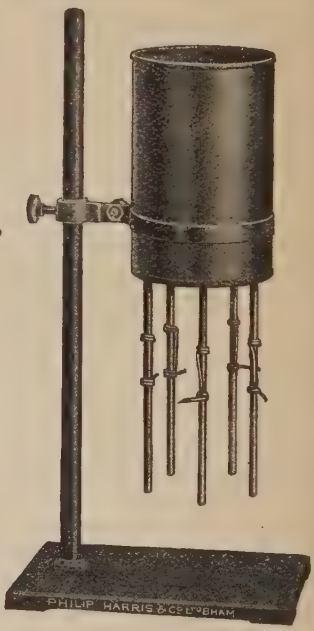

FIG. 91.—Edser's Apparatus.

In Irgenhouz's apparatus (Fig. 90), a number of rods of different metals of the same length and cross-section project from a bath. The rods are initially coated uniformly with wax. On pouring boiling water into the bath, the heat is conducted along the rods at different rates, and these are indicated by the melting of the wax. The length of wax which has been melted is noted at any given instant for all the rods. The difficulty in this experiment is to decide exactly how far the wax has melted. To overcome

this, Edser arranged for the rods to issue vertically from the bottom of the bath (Fig. 91). As the wax melted, a pointed index was able to fall, and the length of wax melted was given at any instant by the position of the index.

Let θ denote the temperature of the bath.
Let φ denote the melting-point of the wax.
Let l denote length of wax melted for any bar after a time T secs.

$$\text{Now } H = \frac{Ka(\theta - \varphi)}{l} T,$$

and since a, θ, φ, and T are constant for all rods, we have

$$K \propto Hl \quad . \quad . \quad . \quad . \quad . \quad \text{(i)}$$

But H, the quantity of heat transmitted, will for any definite pair of temperature-extremes such as θ and φ be greater the greater the length, and hence

$$H \propto l \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

Combining (i) and (ii) we have

$$K \propto l^2.$$

Hence *the relative conductivities of the different rods are obtained by comparing the squares of the lengths of wax melted.*

Absolute Determination of K—Direct Method for Metals.—A good method for the determination of K for good conductors is illustrated in Fig. 92. A bar of the material A is taken, and a difference of temperature is set up between its two ends by supply-

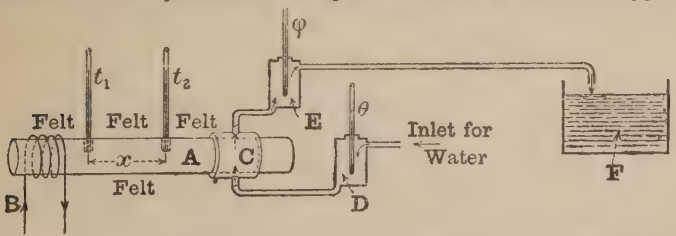

FIG. 92.—Apparatus for finding Conductivity of a Metal.

ing heat at one extremity and extracting heat from it at the other. The heat is supplied either by a steam coil, or, better still, by a coil of wire B, carrying a current of electricity. C is a cooling

water jacket. Cold water is introduced through the thermometer pocket D. It circulates round A at a temperature θ, and after extracting heat from the bar, emerges through the thermometer pocket E at a temperature ϕ. It is then collected in a weighed tank F. The portion of the bar x under test is determined by two small holes containing mercury into which small-size thermometers are introduced, the hot end registering a temperature t_1, and the cool end a temperature t_2.

The whole is enveloped in felt (a poor conducting material) to prevent losses due to radiation, etc.

Suppose m grams of water issue into F in T seconds.

$\therefore$ Heat extracted from the bar $= mc(\phi - \theta)$ calories (c being unity for water).

But when the steady state is reached, heat extracted from the bar = heat transmitted through the bar

$$= \mathrm{K}a\frac{(t_1 - t_2)}{x} \text{calories.}$$

$$\text{Hence } \mathrm{K}a\mathrm{T}\frac{(t_1 - t_2)}{x} = mc(\phi - \theta)$$

$$\text{giving } \mathrm{K} = \frac{mc(\phi - \theta)x}{a\mathrm{T}(t_1 - t_2)}$$

Lees' Method for Bad Conductors.—The method above described is not suitable for bad conductors of heat, on account of the relatively large losses incurred through radiation as compared with the small quantities of heat conducted through the substance. A suitable method for poor, solid conductors is that due to Lees, a simple form of which is shown in Fig. 93. A thin disc of the material is placed between two thin discs of copper or other very good conductor, the contact surfaces being smeared with glycerine to ensure good contact. The heat is either supplied electrically, or, as shown in the figure, by running a constant stream of very hot water through a hollow disc over the copper at one end. At the same time, the other end is kept as cool as possible by a similar stream of very cold water. In each case the good conducting copper discs are at a uniform temperature indicated by special thermometers, and these may be taken as the temperatures of the hot and cold faces of the material under test. The calculation

follows the formula closely, but corrections are necessary for losses due to radiation. The whole forms a very small piece of apparatus, each disc being about 5 cms. in diameter, and perhaps ½ cm. thick. A casing of felt is used to protect against heat losses.

Lees' Method for Liquids.—Lees extended the above method for bad conductors to the determination of K for liquids by

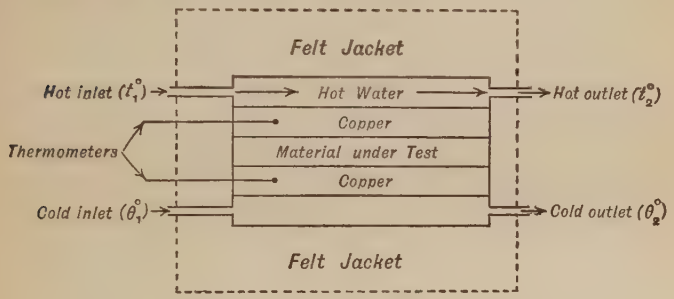

FIG. 93.

substituting for the solid disc of material in Fig. 93 a ring of ebonite and enclosing a thin film of liquid in this ring between two copper discs as before.

Results.—Table IX. at the end of the book gives a series of values of K for a number of solids, liquids, and gases. Generally speaking, the metals are good conductors, and non-metals and liquids poor conductors.

EXAMPLES ON CHAPTER XIII

1. How would you demonstrate that copper is a better conductor of heat than iron, and that mercury is a better conductor than water?
2. What is the thermal conductivity of a substance? How would you compare the thermal conductivities of two metals?
3. A bar of metal is heated at one end; on what does the temperature of a point on the bar depend (1) before and (2) after the bar has reached a steady state?

4. Explain the fact that if a piece of copper gauze is held above a gas-burner and the gas is lighted above the gauze, the gas below the gauze does not ignite. Describe and explain the action of the Davy safety-lamp.
5. Describe a simple method of measuring the conductivity of wood. What source of error is met with in applying the method to metals, and how has it been avoided ?
6. The walls of a cottage are 24 cms. thick, and are of conductivity 0·005. If the temperature of the inside wall is 5° in excess of the outside, and the area of the walls is 200 square metres, how much heat is lost per hour through the walls ?
7. A metal vessel, 1 square metre in area, and whose sides are 0·5 cm. thick, is filled with melting ice, and is surrounded by water at 100°. How much ice will be melted in an hour ? The conductivity of the metal is 0·02, and the latent heat of fusion of ice is 80.
8. A steam pipe at 170° C. is covered with badly conducting material 5 cms. thick. The outer surface is at 30° C. An hourly loss of 45,000 calories is observed per 30 cms. length of pipe, whose mean circumference is 50 cms. Calculate the conductivity of the material.
9. Suppose 10 cms. of ice to have already formed on a pond, and that the air is at −5°. How long approximately will it take for the next millimetre to form ? (Conductivity of ice = 0·005, latent heat = 80.)
10. Describe carefully one method of determining the thermal conductivity of a metallic bar, and enumerate the experimental precautions necessary to secure an accurate result.
11. Define the thermal conductivity of a substance, and explain how it can be measured in the case of badly conducting substances like glass.
12. Define thermal conductivity, and point out some of the difficulties met with in measuring thermal conductivities. Describe some method of comparing accurately the conductivities of substances of low conductivity.
13. Define the coefficient of conductivity for heat. What is the coefficient of conductivity of a badly conducting substance upon which the following experiment was made ? A tin cylinder, 40 cms. in diameter and 50 cms. in length, is covered all over by a layer of the material 0·33 cm. in thickness. Steam is passed through the cylinder at a temperature of 100° C., and the external temperature being 20° C., water is found to accumulate at the rate of 3 grams per minute. The latent heat of steam at 100° may be taken as 537 calories per gram.
14. A pipe of 1·75 inch internal diameter and $\frac{1}{4}$ inch thickness carries hot gases inside, but is surrounded by a cooling

jacket of water, the difference of temperature inside and out being 650° F. It is found that where normally 30,000 units of heat pass through the pipe per hour, when a coating of scale 0·012 inch thick develops inside, only 25,000 heat units pass through per hour. Compare approximately the relative conductivities of the material of the pipe and the scale.

15. A radiator is built up of copper tubes 1·5 mm. in thickness. The mean temperature of the circulating water is 70° C., and that of the air 10° C. The coefficient of conductivity for copper is 0·91 in metric units. How much heat is transmitted to the air per hour per radiator if the effective area of a radiator is 0·248 square metre ?
16. The temperature inside a steel cylinder of 12·5 cms. bore and 14 cms. length is 168° C. The outside temperature is 80° C. The walls are 0·8 cm. thick, and the conductivity coefficient for steel is 0·15. How much heat will pass through the walls of the cylinder per minute ?
17. Ice is to be kept as long as possible in a warm room. Describe the construction of a suitable box, giving reasons for your answer.
18. In tests upon a cast-iron engine cylinder $\frac{3}{8}$ inch thick, it was found that there was a temperature difference of 50° F. between the inner and outer walls. Given that the coefficient of conductivity for cast-iron is 0·0042 B.T.U.'s per square foot per second per degree Fahrenheit drop of temperature per foot of travel, how much heat passes through per square foot of surface per minute ?
19. The inner wall of a vertical engine cylinder of 4 inch bore has a mean constant temperature of 900° F. while the engine is running. Cooling by conduction to the water jacket takes place round the cylinder walls over a height of 8 inches, the wall being $\frac{1}{2}$ inch thick. If the co-efficient of conductivity of the material is 0·005 B.T.U.'s and the mean temperature of the cooling water is 170° F., how much heat is conducted to the water per minute ?

CHAPTER XIV

THE TRANSFERENCE OF HEAT—RADIATION

Radiant Energy.—It was explained on page 164 that radiation is the third mode of transference of heat from a body A to a body B. It is characterised by the fact that there is no intervening material to play any part in the transmission. The fact that A is "hot," in the sense that it is at a higher temperature than that of its surroundings, tells us that its molecules and the electrons composing these molecules are in a state of agitation. This agitation causes waves to be set up in the ether, just as waves would be set up in a still pond if a stone is thrown in; with the difference that the waves in the ether are more complex. A wave is usually made up of a line of vibrating particles, each vibrating as a result of the vibrations of its neighbours, *i.e.* the vibrations are "passed along the line" until something intervenes to prevent their being passed further. In our example the body B is the intervening substance, and as a consequence of the impinging of the ether waves upon it, agitations are set up in the electrons within the molecules of B, and this, as we know, reflects itself in a rise in temperature. We say that B is absorbing the radiations which are being emitted from A. *At* A and B we know the energy to be in the form of *heat*; any instrument which detects heat is able to tell us this. *Between* A and B, however, the energy is no longer in the form of heat. It has been converted to ether waves, and we speak of it as **radiant energy.** *We have no means of detecting radiant energy as such;* there is no such thing as the "temperature of space" about which we so often read. Space implies that from which matter is absent, and where matter is absent, ideas of temperature are non-existent, for temperature measures the degree of warmth of matter. We can only detect radiant energy by converting it into heat energy, *i.e.* by inter-

posing an absorbing body, such as the body B to which we have just referred.

Light and Heat—Electro-magnetic Nature of Ether Waves.—But heat alone is not the only form of energy, which can be converted into radiant energy. Light does this also. Thus the sun not only supplies us with heat, but also with light; and the means by which this is done is identical in the two cases. Both light and heat are a consequence of electronic vibration, and both set up ether waves, and hence both become radiant energy. In what respect then do they differ?

We have pointed out that a wave is usually the "passing on" of a condition of vibration in succession of a line of particles. This is not the only form a wave may take. If we consider a line of points in a medium, then a wave may also mean the "passing on" from point to point of a cycle of changes of any kind in the medium. We have reasons for believing that the ether waves are not of the nature of lines of vibrating particles. The effect of the rotation and oscillations of the minute negative electrons about the positive nucleus of a molecule is to set up a cycle of electro-magnetic changes. It is unnecessary for the student to understand the exact nature of these changes; but it is the propagation through space of this cycle of electro-magnetic changes which constitutes an ether wave. Every point along the "line of wave" passes repeatedly through the complete cycle of changes, just as every particle along an air-wave of sound passes through a cycle of vibrations, and for convenience we may refer to a complete cycle of electro-magnetic changes at a point as a complete "vibration." The student, however, must be careful not to confuse this term with its ordinary significance. Using the term in this sense, therefore, we may say that each ether particle vibrates with a definite *period* which is such a very small fraction of a second that its *frequency*, or number of vibrations per second, amounts to billions. A body which is emitting both heat and light creates waves of thousands of different frequencies in the ether. Beyond this there is no physical difference between heat and light. The vast difference of impression which they convey to us as human beings is solely a consequence of how the nervous system responds to them. If we substitute ourselves for the body B, then the radiant energy falls on us, and we find

that for a certain range of wave frequencies it is the optic nerve that is affected, and we interpret the energy as *light*; whilst for a range of frequencies below this we find the sight is in no way affected. Instead, our sense of temperature interprets the radiant energy for us as *heat*. Roughly speaking, our nerves interpret as light all waves whose frequencies range from (4×10^{14}) per second to (7×10^{14}) per second; below (4×10^{14}) per second our nerves interpret the waves as heat. Heat and light are not the only forms of radiant energy; the others, however, call for no special reference in this book.

The Detection of Heat Radiation—Diathermancy and Athermancy.—We know from experience that many substances are transparent to light; that is, when we interpose such a body (*e.g.* a pane of glass) in the path of radiant energy, part of which may consist of "light waves," the vibrations are not stopped and converted into light, but pass right through the body and continue their course. In exactly the same way we have bodies which are transparent to heat vibrations. Such bodies do not get hot when interposed in the path of "heat waves"; *i.e.* their molecular agitations are not appreciably increased. We speak of them as being **diathermanous,** whilst substances which *do* absorb the radiations and exhibit a consequent rise in temperature are called **athermanous.** Further, just as no body is perfectly transparent to light, so no body is perfectly transparent to heat. Amongst the many thousands of different frequencies of vibration of the different waves, there are always some for which a body will be diathermanous, and others for which it will be athermanous.

In speaking of the waves for which any particular medium is transparent or opaque, we more usually refer to the **wave-length** than to the frequency. *We mean by the wave-length the distance through which the wave as a whole has travelled in the time that any one of the line of "vibrating particles" constituting the wave has completed one entire "vibration."* Obviously the greater the duration of one complete vibration, the greater will be the wave-length, but, on the other hand, the less will be the number of complete vibrations per second, *i.e.* the less will be the frequency. Hence waves of high frequency have small wave-lengths, whilst waves of low frequency have long wave-lengths. **Now, of all substances, perfectly black bodies are the most athermanous;**

practically all radiations are absorbed, both heat and light. When radiant energy falls on such a body, *e.g.* lampblack, it quickly becomes hot and remains black. It becomes hot because all the thermal radiations are absorbed and converted into heat, and it remains black because all the light radiations are absorbed, and none are either reflected back or passed through. For substances other than perfectly black bodies, the degree to which diathermancy obtains is different for different wave-lengths. Thus *glass* is transparent to light waves of wave-lengths roughly between 7×10^{-5} cm. and 3×10^{-5} cm., but is opaque to heat waves longer than 7×10^{-5} cm. wave-length; hence the use of glass firescreens. Ice and distilled water are both very opaque to heat rays; but, on the other hand, *pitch,* which is opaque to light waves, is transparent to a wide range of heat waves, of wave-lengths roughly from 7×10^{-5} cm. to 20×10^{-5} cm. *Quartz,* in contrast to black substances, is transparent to nearly all wave-lengths, not only for those of light and heat, but also for wave-lengths shorter than those to which the eye is sensitive. *Rock-salt* and *sylvite* (potassium chloride) are also transparent over a wide range of heat radiations. Hence, when it is desired to interpose in the path of heat rays, a substance which will permit of the passage of the rays through it, quartz, rock-salt, or pitch would be used. When, however, we wish to *stop* the heat waves, we interpose a black body; this absorbs the radiant energy and exhibits a rise in temperature. By noting this rise in temperature we are able to detect the presence of thermal radiations.

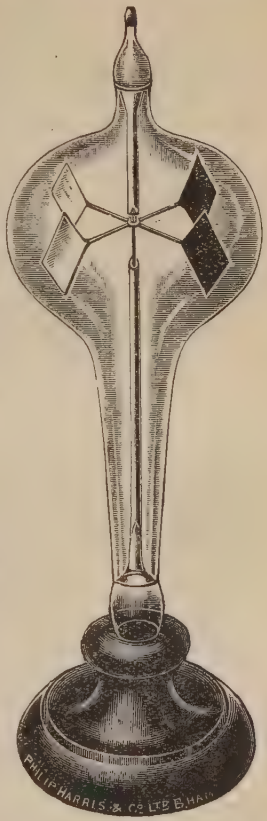

FIG. 94.—Crooke's Radiometer.

A striking illustration of the absorbing power of black surfaces is seen in *Crookes' radiometer* (Fig. 94). This consists of four vanes delicately suspended in a bulb from which *nearly* all the air has been extracted. One side of each vane is blackened, and the other polished, so that as the instrument is turned round the faces are alternately black, bright, black, bright, etc. When heat radiations fall on the instrument, the vanes rotate in a direction such that the black faces are pushed backwards. The explanation is as follows. Of the two faces of each vane, the black face absorbs the radiations, while the bright face reflects them back. Hence the black faces become warmer than the others. This raises the temperature of the air molecules on the black side above that of the molecules on the bright side. And while normally the pressures on the faces as caused by the impact of the molecules upon them are equal, the effect of this excess of temperature on the black side will be to increase the velocity of the molecules, and therefore to cause the pressure on the black face to be greater than that on the bright face. Hence the vanes always revolve with the black faces moving backwards. The reason for removing most of the air from the bulb is to avoid convection currents.

FIG. 95.—Thermoscope.

Fig. 95 shows a simple detector of heat radiations—the *thermoscope*. Exhausted of all air, the instrument contains some ether (in the lower bulb) and ether vapour. The lower bulb is blackened, and when heat rays fall on it, it absorbs them, and its temperature goes up. This increases the vapour pressure of the ether vapour in the lower bulb above the liquid, and it accordingly expands and forces the ether round the bend and up the stem. The extent of this motion is a measure of the amount of radiation which falls on the bulb.

The *differential thermoscope* is simply the usual form of differential air thermometer described on page 8 with the bulbs blackened to increase the sensitiveness of the instrument.

LAWS OF HEAT RADIATION

1. Heat Radiation Travels in Straight Lines.—This can very easily be demonstrated by interposing a screen of a suitable kind in the path of the heat rays. Such a screen usually consists of two tin sheets, placed vertical and parallel to each other, and separated by an air space of about 1 cm. It is found that while a recording instrument placed behind the screen S (Fig. 96) anywhere in the region of the "heat shadow" records no effect, as soon as it is placed outside this region a rise in temperature is at once recorded.

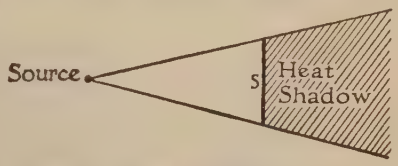

FIG. 96.—Heat Radiation travels in Straight Lines.

Another experiment is illustrated in Fig. 97. S is the heat source, and B is a hole in a double screen placed fairly close to S. Behind B is another screen blackened over with lampblack. Then it is found that while a recording instrument placed anywhere in front of this second screen records no rise in temperature, when placed at A in a line with B and S a definite rise is recorded.

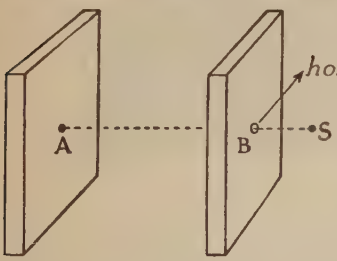

FIG. 97.

2. Heat Rays obey the Same Laws of Reflection and Refraction as Light.—That heat rays are reflected in accordance with the laws of reflection for light can be demonstrated by the apparatus shown in Fig. 98. The reflecting surface is a bright sheet of tin, mounted so that it can easily be turned about a vertical axis through any angle. This angle is indicated by a pointer moving over a graduated arc on the base-board. The heat rays are confined along their paths by two long tin tubes

mounted horizontally so that their axes meet on the screen. The tubes are arranged at angles equally inclined to the pointer. The source of radiation, in the form of a hot metal ball, is mounted so as to be at one end of one tube, and a thermoscope is placed at the corresponding end of the other tube. An immediate and considerable rise in temperature is at once indicated by the thermoscope. If, however, the screen be rotated into any other position, very little rise indeed is indicated. Hence the rays

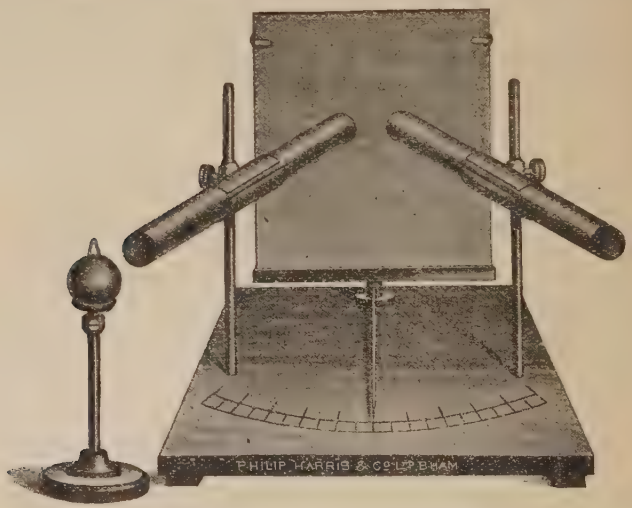

FIG. 98.—Law of Reflection for Heat Rays.

are reflected such that the angle of incidence is equal to the angle of reflection, etc. It should be pointed out that a protecting screen of felt or asbestos is interposed between the hot body and thermoscope to prevent the passage of heat direct from the one to the other. This screen is not shown in the figure.

A further verification may be obtained by arranging two large concave, spherical mirrors of brightly polished copper with their principle axes coinciding, as shown in Fig. 99. The hot body is supported at the focus of one mirror. Heat rays coming from it are reflected, as with light mirrors, into a beam of rays

parallel to the axis. When this beam strikes the other mirror, the rays are again reflected so as to converge on to its focus. Hence a recording instrument placed at this focus at once indicates a rise in temperature, and if a small piece of easily inflammable substance be placed there it quickly catches fire.

That heat rays are refracted in the same way as light rays may also be shown by a number of simple experiments. It will be sufficient here to point out that just as a beam of parallel

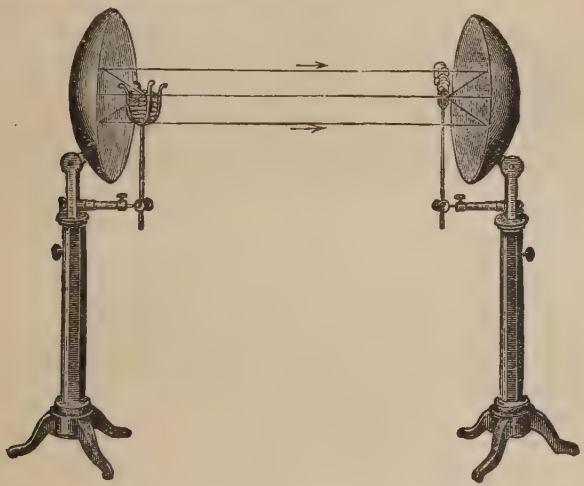

FIG. 99.—Law of Reflection for Heat Rays.

rays of white light is spread out as a result of refraction when it is passed through a triangular prism, so we are able to spread out the rays from a heat source by interposing a prism of quartz, pitch, or rock-salt.

The "burning-glass" familiar to every schoolboy is another example of the refraction of heat rays passing through a lens.

3. **The amount of Radiant-Energy falling upon Unit Surface of a Substance is Inversely Proportional to the Square of the Distance of the Surface from the Heat Source.**—According to the wave theory, heat rays travel outwards from a point-source equally in all directions. Consider a point-source S (Fig. 100)

emitting a quantity of radiant energy Q units per second. Let this energy fall on a sphere of radius r_1.

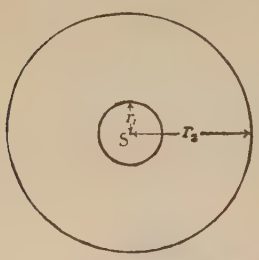

FIG. 100.—Inverse Square Law for Radiant Energy.

Area of spherical surface $= 4\pi r_1^2$.

$\therefore$ Q units per second fall on $4\pi r_1^2$ sq. units of area.

$\therefore$ $\frac{Q}{4\pi r_1^2}$ units per second fall on one unit of area at a distance r_1.

If, however, the rays be intercepted by a sphere of radius r_2, the same amount of energy per second Q will now be distributed over a larger area $4\pi r_2^2$, so that the intensity of radiation, or the amount of heat falling per second upon unit area, will now be $\frac{Q}{4\pi r_2^2}$. This is much less than that falling on unit area of the smaller sphere.

Comparing, we have

$$\frac{\text{intensity of radiation at distance } r_1}{\text{intensity of radiation at distance } r_2} = \frac{\frac{Q}{4\pi r_1^2}}{\frac{Q}{4\pi r_2^2}} = \frac{r_2^2}{r_1^2}$$

Hence *the intensity is inversely proportional to the square of the distance from the heat source.*

Emissive Power.—Of all surfaces, none radiates heat faster than a perfectly black surface. **We speak of the emissive power of a surface as the amount of heat it radiates per sq. cm. per second compared with the amount of heat radiated by a perfectly black surface per sq. cm. per second.**

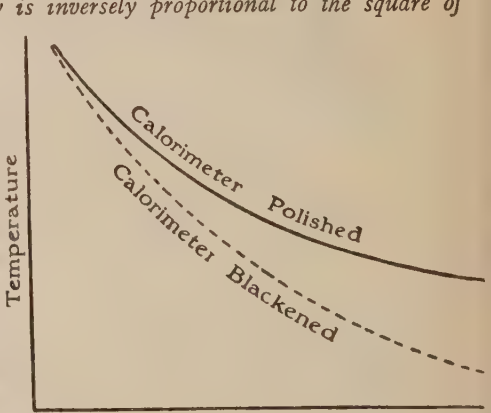

FIG. 101.—Influence of Surface on Emission of Thermal Radiations.

An obvious method for determining this is by a comparison of cooling curves (Fig. 101). For a given small temperature range, the emissive powers will be inversely proportional to the times of cooling; or, for a given time, it will be directly proportional to the fall in temperature.

Leslie, in 1804, used the apparatus shown in Fig. 102 for comparing the emissive powers of different surfaces. A square tin vessel A, filled with boiling water, had its vertical faces coated over with the various substances to be compared. The heat rays emitted by any one face were intercepted by a parabolic brightly polished tin mirror B, and were reflected to a focus at C. The recording instrument was placed at C.

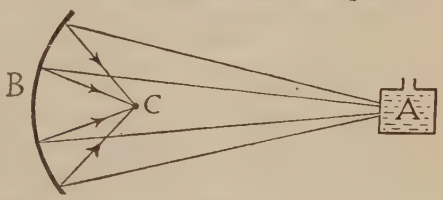

FIG. 102.—Leslie's Method of Comparison of Emissive Powers.

De la Provostaye and *Desains*, using an improved method, obtained the following results when the temperature of the radiating body was at 120° C.:

Lampblack . . .	100	Copper . .	5
Indian ink . . .	88	Silver . . .	2 to 3
Platinum . . .	11		

It is therefore seen that whilst a perfectly black surface acts as a "complete radiator," the brighter the surface the smaller is its emissive power. For the polished metallic surfaces the value is very low indeed.

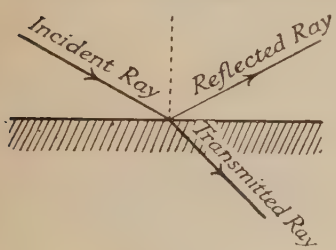

FIG. 103.

Reflecting Power. — When a "bundle" of heat rays is incident upon a surface, the energy divides up into three portions. Part of it is reflected back into the first medium (Fig. 103) in a direction such that the angle of incidence is equal to the angle of reflection.

A second portion is transmitted through the medium, *i.e.* to these rays the medium is diathermanous; and the remainder, which is neither transmitted nor reflected, is absorbed by the medium, and produces a rise in its temperature.

It is natural to expect that a surface from which the heat is largely reflected will absorb very little heat, whilst a substance which is a good absorber will necessarily reflect very little heat.

The reflecting power of a substance is defined as the amount of heat which is reflected from its surface per sq. cm. per second compared with the amount of heat which is incident upon it per sq. cm. per second.

Leslie compared the reflecting powers of various surfaces by a method which was a slight modification of his method for comparing emissive powers described in the previous article. Rays from a heat source S (Fig. 104) were reflected from a large metal mirror, and instead of being brought to a focus at C, were intercepted by the surface AB whose relative reflecting power it was desired to find. The rays which were reflected from AB were brought to a focus at T and were there made to affect a suitable recording instrument. Another surface of the same dimensions as AB was now substituted, and the experiment repeated. The ratio of the readings was the ratio of the reflecting powers. The following were some of the values found :

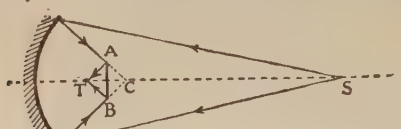

FIG. 104.—Comparison of Reflecting Powers.

Fraction of Incident Light.

Silver	0·97	De la Provostaye and Desains.
Gold	0·95	
Brass	0·93	
Speculum metal	0·86	
Tin	0·85	
Steel	0·83	
Glass	0·10	Leslie.
Lampblack	0·00	

Absorptive Power.—We define as the absorptive power of a surface the ratio of the amount of heat absorbed per sq. cm. per second to the amount of heat falling on it per sq. cm. per second.—The absorptive powers of different substances have been compared by *De la Provostaye* and *Desains* by the following method. The bulb of a thermometer was coated with the material to be tested, and was then heated to a temperature higher than that which would subsequently be used for experiment. It was then allowed to cool in a protected vessel, and a cooling curve was drawn up. The thermometer was next exposed to heat rays from some suitable source, the rays being directed on to the bulb by means of a "heat lens." At first the thermometer was receiving heat faster than it could radiate it, and a rise in temperature was accordingly recorded. After a time, however, the mercury thread remained steady, showing that the heat absorbed was now equal to the heat radiated, *i.e.* the steady state was reached. Suppose the temperature at which this happened was $\theta°$, and that a reference to the cooling curve previously drawn showed that the rate of cooling corresponding to this temperature was θ' per second. Then denoting by W the water-equivalent of the bulb, the heat lost per second was $W\theta'$. But since the steady state was reached, $W\theta'$ must also represent the heat absorbed per second. Using the same source of heat, the experiment was now repeated with a different substance covering the bulb, and a new value $W\theta''$ was obtained for the heat absorbed per second. Hence the ratio of absorbing powers

$$= \frac{\text{Heat absorbed by first surface}}{\text{Heat absorbed by second surface}}$$

$$= \frac{W\theta'}{W\theta''} = \frac{\theta'}{\theta''}$$

The following are some of the results which have been obtained when the heat source was at 100° C.:

Lampblack	100
Isinglass	91
Indian ink	85
Shellac	72
Metals	13

The results are very interesting, for comparing them with those obtained for the emissive powers of surfaces, it is seen that **a substance which is a good absorber of heat is a good radiator of heat, whilst a substance which is a bad absorber of heat is a bad radiator of heat.** Also, *a substance which is a good absorber or radiator is a poor reflector, whilst a good reflecting surface is a poor radiating or absorbing surface.*

Ritchie's Experiment.—The relationship between emission and absorption has been very strikingly illustrated by Ritchie in an experiment shown diagrammatically in Fig. 105. An air thermometer of the differential type has two drum-shaped bulbs, the inner surface A of one being lampblacked, whilst the inner surface of the other D is a polished white surface. Interposed between these is a cylindrical can containing boiling water at 100° C. The end B facing the blackened surface A is bright and polished, whilst the other end C facing the bright face D is blackened.

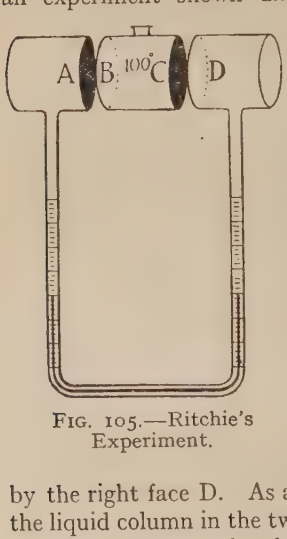

FIG. 105.—Ritchie's Experiment.

The white face B emits very little radiation, but such as is emitted is readily absorbed by the black face A. The black face C, on the other hand, emits a good deal of radiation, very little of which, however, is absorbed by the right face D. As a result, it is found that the readings of the liquid column in the two stems are the same as before, showing that the total heat absorbed by D is equal to the total absorbed by A.

Let E = emissivity of surface C of the cylinder.

,, a = absorptive power of surface D of the right drumhead.

∴ amount of heat absorbed by D per sq. cm. per second = Ea.

Also, let e = emissivity of surface B of the cylinder, and A = absorptive power of surface A of the left-hand drumhead.

∴ Amount of heat absorbed by A per sq. cm. per second = eA.

But since the levels indicate no change, plates A and D absorb equal amounts.

$$\therefore \quad eA = Ea$$

$$\therefore \quad \frac{e}{E} = \frac{a}{A}$$

∴ The ratio of the emissivities of the two surfaces is equal to the ratio of their absorptive powers.

Prévost's Theory of Exchanges.—The various facts which we have considered on the subject of radiation are all consistently explained by the *Theory of Exchanges* initially propounded by *Prévost*, of Geneva. When a hot body is placed in cooler surroundings, we know that it radiates its heat to them, its temperature diminishing in consequence, until it has been reduced to the temperature of the surroundings. Apparently nothing further now takes place, and the body has ceased to radiate heat. A moment's consideration will show, however, that this is impossible, unless we reject the kinetic theory of matter and the wave theory of radiant energy.

Waves are propagated through the ether, it will be remembered, by the vibrations of electrons. Only if these electrons are at rest will it be impossible for ether waves to exist. We know, however, that so long as a body is at a temperature above that of the absolute zero, *i.e.* above $-273°$ C., its molecules, and therefore its electrons, are not at rest, but are vibrating, and therefore generating radiant energy in the form of ether waves. Further, to each electron there is "attached" a definite period of vibration, thus giving rise to waves of definite frequency and wave-length. With the millions of molecules constituting any portion of matter, there are no doubt thousands of different periods of vibration, and therefore such a small body will generate waves of thousands of different frequencies.

What happens when these waves, having travelled through the ether, are intercepted by another collection of millions of molecules in the form of another small body? Here there comes into play what is known as the *principle of resonance.* Each wave instinctively "picks out" that electron whose frequency of vibrations coincides with its own, and if that electron is not already vibrating, the wave sets it vibrating "in tune" with its own period, whilst if, as will in general be the case, the electron

is already vibrating with that frequency, it is set vibrating more violently; so that its *amplitude*, *i.e.* its range of oscillation, is increased. The ether wave will not have the slightest effect on any other electron whose frequency differs from its own. Hence the radiant energy which was in the form of ether waves is now converted into molecular energy, *i.e.* into *heat*, and this is manifested, as we know from the kinetic theory, by a rise in temperature. We say these waves are *absorbed* by the body. Again, suppose it happens that amongst the waves which are intercepted by the body there are those of certain frequencies such as are possessed by no electron in the intercepting body. These waves will be unable to influence the molecules or their electrons in any way whatever, and they cannot be converted into molecular energy or heat. They will therefore pass on right through the body and continue their course through the ether beyond, *i.e. to such waves the body will be transparent.*

It will now be obvious to the student that perfectly black bodies are such as possess molecules containing electrons which will vibrate with every known variety of frequency. Such a surface can therefore generate every variety of wave-length, and when intercepting radiant energy, is able to absorb every variety of wave-length impinging on it. For the same reason, no waves will be able to penetrate the medium. It is perfectly opaque. Bodies, on the other hand, which are not perfectly black, are lacking to different degrees in electrons of certain frequencies of vibration.

Such surfaces are therefore unable to generate corresponding waves, and if waves with such frequencies impinge upon them, they cannot be absorbed, but must pass through. The brighter the body, to a greater degree does this happen. Finally, it should now be perfectly evident why a body which is a good radiator is a good absorber, whilst a body which is a bad radiator is a bad absorber. For both are governed by the one factor of the ability of the electrons constituting the body to vibrate in a definite manner. All these considerations led *Prévost* to the conclusion that when a body has cooled to the temperature of its surroundings, it is incorrect to assume that radiation has ceased. The fact that its electrons are still vibrating proves that radiation, *i.e.* the generation of ether waves, still continues, but on the other

hand the argument applies equally to the surroundings, the electrons of whose molecules must be at the same time radiating heat to the body. An exchange is therefore going on, and the temperature of the body now remains constant merely because the amount of heat which is being radiated *to* the surroundings is exactly equal to the amount of heat which is being absorbed or received *from* the surroundings. A state of thermal equilibrium thus exists. This not only applies to radiations in general —it is true for every particular wave-length, for we can envelop the body with a screen such as will only be transparent to one particular wave-length, and still the temperature remains the same. The theory of exchanges therefore holds for radiations of particular wave-lengths as well as for radiations in general.

EXAMPLES ON CHAPTER XIV

1. Explain the term radiant energy. Give an account of the diathermancy of glass for radiation from different sources.
2. Why is white clothing worn during summer?
3. How does the intensity of radiation at a point due to a small source depend upon the distance from the source? How may an experimental test of the law of radiation be made?
4. How do you account for the fact that on a frosty night it is often colder at the bottom of a valley than on the neighbouring hillsides?
5. How can it be shown experimentally that the heat radiation from a hot body obeys the same laws of reflection and refraction as light?
6. Upon what does the rate at which a body radiates heat depend? Prove from general principles that the radiating power of a body is equal to its absorbing power, and describe an experiment which illustrates this relation.
7. Describe how experiments may be made to measure the property for solids of reflecting, absorbing, and transmitting radiant heat.
8. Explain Prévost's theory of exchanges, and point out its application to the relation between emissivity and absorbing power of a substance.
9. Give theoretical and experimental evidence for the statement that the radiating and absorbing powers of a body are proportional to one another.

CHAPTER XV

THE CONSERVATION OF ENERGY—THE MECHANICAL EQUIVALENT OF HEAT

We have several times during the course of this book referred to heat as a form of energy. It is proposed in this chapter to discuss this subject and its consequences more fully.

The First Law of Motion-Force.—The natural condition of a body with respect to its surroundings is one of rest or of uniform motion in a straight line ; that is, if it were possible for a body to be isolated in space from all external influences, then either it would be at rest, or it would be moving in a straight line with uniform velocity. We have no direct proof of this, for it is impossible so to isolate a body, but as a matter of common sense we know that if such a body is at rest, it will remain so because there is no reason whatever why it should start moving in any way ; if, on the other hand, the body is moving, there is no reason whatever why the manner of its motion should alter in any way, so that it would continue in a straight line with uniform velocity. It is contrary to our instinct and experience to expect a body at rest suddenly and of its own accord to start moving, or, if it *is* moving, suddenly to alter its speed or its direction. This inability of a body so to control its own state of rest or motion is spoken of as the *principle of inertia.*

When, therefore, a body does change its state of rest or of uniform motion in a straight line, it can only be due to the action of some external influence. We speak of such an external influence as **force.** *Force is therefore defined as that which, acting on a body, changes or tends to change its state of rest or of uniform motion in a straight line.* Both the principle of inertia and the definition of force are merely alternative forms of a statement known as Newton's First Law of Motion, which says that all

bodies continue in a state of rest or of uniform motion in a straight line, except in so far as they are prevented from so doing by the action of external forces.

Work and Energy.—We see therefore that the tendency of force is to change motion.[1] A body which is endowed with motion is said to possess "energy." Such energy is extremely useful, and it is by creating it in bodies by the application of forces of various kinds that the conveniences and comforts of civilisation are made possible. Thus by bringing suitable forces into play, a heavy train is able to move with a high speed and to carry food and people from one place to another; or we are able to take advantage of the force of the wind to turn the sails of a windmill, and so to grind corn.

When a force, acting on a body, causes it to move, we say that work is done by the force on the body, and we measure work by the product of the force, and the distance through which it acts; *i.e.* if F is the force and *s* is the distance it moves, the amount of work (W) done by the force is

$$W = Fs.$$

The usual unit of work is the *foot-poundal*[2] in the F.P.S. system, and the centimetre-dyne or *erg* in the C.G.S. system. Suppose a force acts upon a body so as to cause it to move, and that the force is then withdrawn. The body is now endowed with a certain amount of energy, supplied by the force, and the measure of this energy is the amount of work the body is capable of doing. Since the force is withdrawn, and the amount of energy imparted to the body is limited, as the body moves it will gradually be expending this energy, *i.e.* the amount of motion left in it is constantly diminishing. Eventually it will all have been expended, and the body will come to rest. It has "given up" all the energy which was imparted to it by the force.

[1] Two bodies with the same velocity, but having different masses, can hardly be said to possess the same amount of motion, *e.g.* a tramcar has a much greater motion than a bicycle moving with the same speed. Hence we measure motion by the product *mass* × *velocity*, and we speak of the product as momentum.

[2] From the *poundal*, the unit of force, equal to $\frac{1}{32}$ of a pound weight. The corresponding force unit in the C.G.S. system is the *dyne*, equal to $\frac{1}{981}$ of a gram weight. The engineer uses the *pound weight* as the practical unit of force, and so his unit of work is the *foot-pound*.

It may be shown that if a body is at any instant moving with a velocity v, the amount of *mechanical energy* (as we call the energy of motion, to distinguish it from other forms of energy to which reference will presently be made) it possesses and is giving up as it is moving is equal to $\frac{1}{2}mv^2$ foot-poundals (or ergs), where m is the mass of the body in pounds (or grams).

If such a body be opposed by a force F, its motion will gradually be reduced, and eventually it will be brought to rest. It will have lost all its mechanical energy, but in causing this the force F must have been doing an amount of work equal to the energy extracted from the body. If the distance through which the force moves to bring the body to rest be s, then we have

$$Fs = \tfrac{1}{2}mv^2 \text{ foot-poundals (or ergs).}$$

Example 1.—*A bullet whose mass is* 5 *oz. is obstructed by a pane of glass* 1 *inch thick at a moment when it is travelling with a velocity of* 1000 *feet per second. If the bullet emerges with a speed of* 700 *feet per second in consequence of the resistance offered by the glass, find the amount of this resistance.*

Solution.—Assuming there are no other factors involved, we have

work done by resistance = loss in energy of the bullet.

$$\therefore \quad F \times \tfrac{1}{12} = \tfrac{1}{2}m1000^2 - \tfrac{1}{2}m700^2$$
$$= \tfrac{1}{2}\tfrac{5}{16}(1000^2 - 700^2)$$
$$\therefore \quad F = \tfrac{5}{32} \times 300 \times 1700 \times 12$$
$$= 956{,}250 \text{ poundals.}$$

It should be noted that in order to convert foot-poundals to foot-pounds we must divide by g $(=32)$, *e.g.*

$$\text{kinetic energy} = \tfrac{1}{2}mv^2 \text{ foot-poundals}$$
$$= \frac{\tfrac{1}{2}mv^2}{g} \text{ foot-pounds.}$$

The Two Forms of Mechanical Energy.—The fact that a body is in motion is definite evidence that it possesses mechanical energy. Nevertheless it is possible for the body to possess such energy even when it is at rest. Thus, suppose it is lifted from the floor of a room, and placed at rest just at the edge of a shelf

near the ceiling. The slightest displacement will suffice to cause the body to fall to the floor again. This proves that in its position on the shelf the body is possessed of a store of energy, since in falling it does work, *i.e.* it gives up the store of energy. The amount of energy so stored is equal to the amount of work it does in falling back to the floor. Hence we have

$$\begin{aligned}\text{Work} &= \text{force} \times \text{distance} \\ &= \text{weight} \times \text{height of fall } (h) \\ &= mgh \text{ foot-poundals (or ergs).}\end{aligned}$$

Hence the energy stored up in a body at rest at an elevation of h above the ground is equal to mgh units. *There are therefore two forms of mechanical energy:* (1) *the energy which a moving body possesses by virtue of its speed, and* (2) *the energy which a body possesses by virtue of its position at an elevation above the earth's level.*

The former is known as **kinetic energy,** the latter as **potential energy.**

It should be noticed that a body need not be at rest in order to possess potential energy. For if it be moving with a velocity v at an elevation h, then apart from its kinetic energy equal to $\frac{1}{2}mv^2$, it possesses an additional store of potential energy equal to mgh, so that its total mechanical energy $= \frac{1}{2}mv^2 + mgh$.

Relation between Kinetic and Potential Energy of a Falling Body.—The case to which we have just referred of a body falling from a shelf will repay further consideration. If its height above the floor is h, then its store of potential energy is mgh units. When, as a result of a slight displacement, it commences to fall, it does so under the influence of the *force of gravity*. The direction is downwards because the direction of the earth's gravitational force is downwards, *i.e.* towards the centre of the earth. Now the effect of a force upon a body is to increase its motion, or, in other words, to produce an acceleration in the body. Indeed, we measure the force by the acceleration it produces, the analytical definition of force being $F = ma$, where m is the mass of the body, and a the acceleration produced. As the body falls from the shelf, therefore, its velocity increases (at the rate of 32 feet per second per second or 981 cms. per second per second), so that it acquires more and more kinetic energy the nearer it approaches the

ground. But, on the other hand, as the distance from the floor diminishes, the potential energy must also diminish ; **as the body gains in kinetic energy it loses in potential energy.** It is easily shown by elementary mechanics that for such a falling body

kinetic energy + potential energy = constant.

The same applies to a body thrown up into the air. The force of gravity opposes the motion, and reduces the velocity until the body reaches its highest point. Hence as it rises its kinetic energy diminishes, but its potential energy increases, the loss in the one being compensated by a gain in the other.

Sources of Energy—Heat.—This relationship between kinetic and potential energy is an interesting example of one of the most important facts of nature. We have seen that no inanimate body can endow itself with energy. If a body is moving, the kinetic energy it possesses must have been supplied by some external agency, and the same is true for potential energy. The body which is lifted from the floor to the shelf receives its energy from the lifting force—the hand, for example, and the hand receives its energy from the muscles. Or if a body is lifted by some mechanism, then the energy is supplied by the mechanism. But that which applies to the body applies to the mechanism, which also cannot endow itself with energy. No machine works itself, although it might appear to do so. Energy must therefore also be supplied to the machine, as by turning a handle.

Think of the vast amount of machinery used all around us, and of the vast stores of energy thus set free. Where does it all come from ? It is beyond our instinct and common sense to imagine that it creates itself. Take an ordinary engineering workshop worked by a steam-engine as a typical example. The various machines—lathes, milling machines, planing machines, etc.—are worked by means of a driving belt which is driven round by shafting suspended from the workshop ceiling. This shafting is driven round by means of a rotating fly-wheel to which it is geared. The fly-wheel goes round because it is attached to a rotating crank-shaft, and the crank is made to rotate by means of a connecting-rod and cross-head moving backwards and forwards. This in its turn is moved by a piston fitting into a cylinder, whilst the piston is pushed backwards and forwards by the power of

expanding steam. We have therefore traced the energy possessed by the machinery to the steam in the cylinder of a steam-engine. And whence does the steam derive its energy? Obviously from the source which created it, *i.e. the heat in the fire-box under the boiler.*

Let it not be forgotten that steam is not the only motive power used in engines. The piston is often made to move to and fro by the combustion of gases and oils; but, again, *heat is the source of the energy.* Just as it is impossible for energy to create itself, so it is impossible for energy to disappear into "nothingness"; this also is contrary to common sense. The energy which is given up by the hand in lifting the body is transferred to the body as potential energy, and, when it falls, the potential energy given up is transferred into kinetic energy of motion. Everyone knows how *rubbing produces heat.* When it is very cold we rub our hands together; we transfer the kinetic energy of the motion of the hands into heat, and so we feel warm. Friction, the most universal of phenomena, is a continual example of this; we mean by friction the rubbing of one group of particles against another group which occurs whenever a tendency to motion takes place; and it is in fact merely the conversion of mechanical energy into heat energy. When a body is dropped the sudden stoppage which takes place on reaching the ground is simply the sudden conversion of the kinetic energy into heat; the body is heated. The student has only to rub his hand firmly up and down his sleeve a few times to feel this for himself, and to a very uncomfortable extent if he performs the experiment too zealously. Any engineering student who has worked a lathe knows how hot the shavings feel immediately after they are produced, and how hot the cutting tool becomes. We all know how hot machinery becomes with much work, and how necessary it is to prevent this from being carried to excess by the use of suitable cooling devices.

Nature of Heat Energy.—It appears to be indisputable therefore that mechanical energy is not the only form of energy. **Heat is a form of energy.** What, however, is the nature of this heat energy? The explanation is furnished by the kinetic theory of matter. It was pointed out in Chapter V. that the molecules which constitute matter, and the electrons which constitute the molecules, are not at rest, but are in constant and rapid motion,

and that this motion is continually being interfered with and altered by collisions. Obviously, therefore, even though the body as a whole appears to be at rest, there is a great store of mechanical energy within it, and we speak of this as *internal energy* to differentiate it from the external energy of the body as a whole. This internal energy is not only the kinetic energy due to the motions of the molecules; it is partly the potential energy due to their positions relative to each other. For it will be remembered that the molecules exert attractions upon each other, and in separating molecules work must be done against these attractions, and hence such separation represents a store of potential energy. In solids and liquids, where the molecules are naturally close together, the potential energy forms a considerable proportion of the total internal energy, but in gases the molecules are so far apart relatively that the internal energy is nearly all kinetic, the potential energy being very small. We also learnt from the kinetic theory that when the temperature of a body is increased, the molecules vibrate more rapidly and their mean free path is increased. This, then, is the particular form taken by the transformation of the external energy of a body into heat; *the external energy of motion is converted into the internal energy of molecular vibration.*

Other Forms of Energy.—Mechanical energy and heat are not the only forms of energy in the universe. As was pointed out in the last chapter, that portion of the molecular and electronic vibrations which we interpret as heat is governed entirely by a range of vibration-frequencies within definite limits. We interpret frequencies beyond this limit as light, and as this is the only respect in which light differs from heat, it is obvious that light is also a form of energy. But even the light-vibrations do not exhaust all the vibration frequencies. Beyond the "light range" are vibrations invisible to the eye; these are the ultra-violet or "actinic" rays," so called because of certain chemical actions to which they give rise under certain circumstances. All these vibrations create ether waves of corresponding frequencies, whose values determine the conversion of radiant energy during transmission from one body to another body, as explained in Chapter XIV., into heat, light, or actinic energy.

Other forms of energy are sound, strain, chemical energy, magnetic energy, and electrical energy. It is unnecessary to give

separate accounts of these, as they concern us relatively little in this book.

The Conservation of Energy.—We have already emphasised the fact that it is contrary to experience and to common sense to suppose that energy can in any way either appear of itself from "nothingness" or disappear into "nothingness." Whenever energy appears in any one form, such appearance will always be explained by a corresponding disappearance from another body or another form; similarly, if energy disappears in any form, a corresponding appearance will be found to occur simultaneously in another form. This is a consequence of the **Principle of the Conservation of Energy** which states that **the sum total of the energy of the universe is a constant.** Many of the conveniences of civilisation are the practical outcome of this most important law. Thus, to take a common example, in every large city we have a system whereby, beginning with heat energy in the form of fuel, a transformation is effected by means of a steam or gas or oil engine into mechanical energy; by this means a dynamo is driven, converting the mechanical energy into electrical energy. This energy is transmitted by wires to such places as is required in order to carry out further transformations. Part of it is carried to "electric motors" underneath trámcars, and is thus reconverted into mechanical energy. Part is carried to the various houses for purposes of illumination, and is changed into light energy; and throughout the entire series of changes part is constantly dissipated in the form of heat in the wires and in the mechanisms employed. Where there are waterfalls it is unnecessary to start with heat energy. The far less costly potential energy of the water at the top of the fall is converted into kinetic energy of rotation of the wheels of turbines. For this reason we find large numbers of factories in the vicinity of waterfalls. Such factories need not be directly adjacent to the falls, owing to the ease with which electrical energy can be transmitted for many miles along wires.

The Degradation of Energy.—Of all forms of energy, heat is in itself the least useful. It is true that vast stores of fuel are required every day for the purpose of conversion into other forms of energy utilised in the many phases of daily activity. A moment's consideration, however, will show how much of this heat energy is unavoidably wasted. Whatever operations are

carried on, heat is constantly produced. If we carry a current along a wire, some of the electrical energy disappears in the form of heat, and in certain circumstances it is possible for the wires to become hot to a dangerous degree. In every mechanism of any kind, the working is always accompanied by the production of heat, but for which an equivalent amount of energy would have been available for carrying on the work for which the mechanism is intended. Thus much energy is therefore lost. Every known form of energy "pays its toll" in this way; it is unavoidable in spite of the most skilful efforts of our engineers. The converse change, too, brings us the same result. However much we try to convert heat energy into the more useful forms, complete success is impossible. Think of the tremendous amount of heat energy which is carried off up the flues and chimneys and out into the air, completely lost to utility. Only a small fraction of the fuel consumed is usefully employed. All the rest is dissipated into space. Again, of the vast stores of solar heat energy radiated out into space, only about $\frac{1}{2000000000}$ of it is received by the earth. What happens to the remainder?

So we get the *Principle of the Degradation of Energy*, which tells us that *though the total quantity of energy in the universe is a constant, yet it persistently becomes less and less available for useful work.*

The Mechanical Equivalent of Heat (J).—The Principle of the Conservation of Energy is necessarily quantitative in character. It states that whenever a definite quantity of mechanical energy is expended in the production of heat, an equally definite quantity of heat must appear in consequence. Careful experiments fully bear this out. Of course the units of both work and heat were arbitrarily adopted long before their connection was realised, and so we can scarcely expect the equivalent quantities of the two forms of energy to be numerically equal. As a matter of fact, it is found that **in order to produce one calorie of heat, an amount of work equal to $4{\cdot}186 \times 10^7$ ergs must be performed.** We speak of this as the **Mechanical Equivalent of Heat,** and it is denoted by the letter **J,** after *Joule* of Manchester (1818–1889), who was the first to obtain a really reliable value. *The corresponding value in the British system of units is* 778 *foot-pounds of work per B.T.U.*

Example 2.—*An engine works at 7 horse-power (1 horse-power = 33,000 foot-pounds of work per minute). How much heat is required per minute to produce this, assuming that none is lost? (J = 778 foot-pounds per B.T.U.)*

Solution.—Work done per minute $= 7 \times 33{,}000$ ft.-lb.
$= 231{,}000$ ft.-lb.

1 B.T.U. produces 778 ft.-lb. of work

$$\therefore \text{ Amount of heat required} = \frac{231{,}000}{778}$$
$$= 297 \text{ B.T.U.'s per minute.}$$

Example 3.—*An engine working at 5 horse-power is employed for melting ice at 0° C. How much ice will be melted in 3 hours, assuming that 30 per cent. of the energy is lost in friction, etc. (J = 1390 foot-pounds per pound-degree-Centigrade, and L = 80.)*

Solution.—

Rate of working $= 5 \times 33{,}000$ ft.-lb. per minute.

$\therefore$ Work done per 3 hours $= 5 \times 33{,}000 \times 60 \times 3$ ft.-lb.

$\therefore$ Amount utilised in melting ice $= 5 \times 33{,}000 \times 60 \times 3 \times 0{\cdot}7$ ft.-lb.

$$\therefore \text{ Amount of heat generated} = \frac{5 \times 33000 \times 60 \times 3 \times 0{\cdot}7}{1390} \text{ lb.-deg.-C.}$$

But 80 pounds-degrees-Centigrade of heat melt 1 lb. of ice.

$$\therefore \text{ Amount of ice melted} = \frac{5 \times 33000 \times 60 \times 3 \times 0{\cdot}7}{1390 \times 80}$$
$$= 187 \text{ lb.}$$

Example 4.—500,000 *B.T.U.'s of heat are required per day to heat a house by central heating. The coal used has a heat or calorific value of* 13,000 *B.T.U.'s per lb., and costs* 42*s. per ton. The efficiency of the furnace is* 60 *per cent. Find the cost per week.*

Solution.—500,000 B.T.U.'s represent $\frac{500000}{13000}$ lb. of coal.

But this is only 60 per cent. of the coal supplied to the furnace per day.

$$\therefore \text{ Coal used per day} = \frac{100}{60} \times \frac{500000}{13000} \text{ lb.}$$

$$= \frac{2500}{39} \text{ lb.}$$

$$\therefore \text{ Number of tons used per week} = \frac{2500 \times 7}{39 \times 2240}$$

$$\therefore \text{ Cost per week} = \frac{2500 \times 7}{39 \times 2240} \times 42 = 8\text{s. } 4\text{d.}$$

Example 5.—*A train has a mass of 200 tons, and reduces speed from 60 to 40 miles per hour by the application of the brakes. Assuming that the whole of the work done against the frictional resistance of the brakes is converted into heat, how much heat is developed?*

Solution.—60 miles per hour = 88 feet per second.
and 40 miles per hour = $88 \times \frac{2}{3}$ feet per second.

Now kinetic energy = $\frac{\frac{1}{2}wv^2}{g}$ ft.-lb.,

where w = weight of moving body in pounds.

Hence initial kinetic energy of train = $\frac{\frac{1}{2} \times 200 \times 2240 \times 88^2}{32}$

Also final kinetic energy of train = $\frac{\frac{1}{2} \times 200 \times 2240 \times (88 \times \frac{2}{3})^2}{32}$

$\therefore$ Loss in energy due to application of the brakes

$$= \frac{\frac{1}{2} \times 200 \times 2240 \times 88^2}{32} - \frac{\frac{1}{2} \times 200 \times 2240 \times (88 \times \frac{2}{3})^2}{32}$$

$$= \frac{200 \times 2240}{2 \times 32} \left\{ 88^2 - \left(\frac{88 \times 2}{3} \right)^2 \right\} \text{ ft.-lb.}$$

$$= 7000 \times \left(88 - \frac{88 \times 2}{3} \right) \times \left(88 + \frac{88 \times 2}{3} \right) \text{ ft.-lb.}$$

$$= 7000 \times \frac{88}{3} \times \frac{88 \times 5}{3} \text{ ft.-lb.}$$

Now J = 778 ft.-lb. per B.T.U.

$\therefore$ Heat generated by application of brakes

$$= \frac{7000 \times 88 \times 88 \times 5 \times 1}{3 \times 3 \times 778}$$

$$= 38{,}220 \text{ B.T.U.'s approximately.}$$

Experimental Determination of J.—We have already referred in the opening pages of this book to the experiments performed by Rumford and Davy at the end of the eighteenth century upon the relation between heat and work. It was not, however, until 1840 that accurate experiments were devised for the determination of the mechanical equivalent of heat. Indeed no speculations on the doctrine of the conservation of energy could have any final value until this relationship could be accurately established, and for this we are indebted to James Prescott Joule, the young Manchester brewer and scientific amateur to whom we have already frequently referred. This remarkable young man had spent years in studying the relations between the various forms of energy, his researches culminating in that famous experiment in which he converted the mechanical energy of a falling weight into the heat produced by the rotation of a baffle-plate in a vessel of water, giving him the result of 788 foot-pounds of work as the mechanical equivalent of the pound-degree-Fahrenheit.

Yet no one would listen to Joule. In April 1847 he gave a popular lecture on the subject in Manchester. Most of the papers ignored it, the *Manchester Guardian* being a notable exception; and it remained for *William Thomson*, afterwards *Lord Kelvin* (1824–1907), to force recognition for Joule from a reluctant world of science. Joule was due to read a paper on the subject of his experiments before the British Association at Oxford in June 1847, but the Chairman, to whom the paper seemed unimportant, asked him to be brief. "I can never forget," said Thomson, when as Lord Kelvin he was unveiling a statue of Joule in Manchester in 1893, "the British Association at Oxford in 1847, when . . . I heard a paper read by a very unassuming young man, who betrayed no consciousness in his manner that he had a great idea to unfold. I was tremendously struck with the paper. . . ." As to what happened on the conclusion of his reading, we may turn to Joule's own account. "Discussion not being invited, the communication would have passed without comment, if a young man had not risen in the section, and by his intelligent observations created a lively interest in the new theory. The young man was William Thomson."

EXPERIMENTAL DETERMINATION OF J.

Joule's Experiment.—Joule's apparatus is shown diagrammatically in Fig. 106. The principle is one common to a large number of laboratory methods for finding J, namely, the creation of heat by churning up water in a calorimeter. Joule used a suitably protected calorimeter containing a known quantity of water. Immersed in this water was a system of eight brass

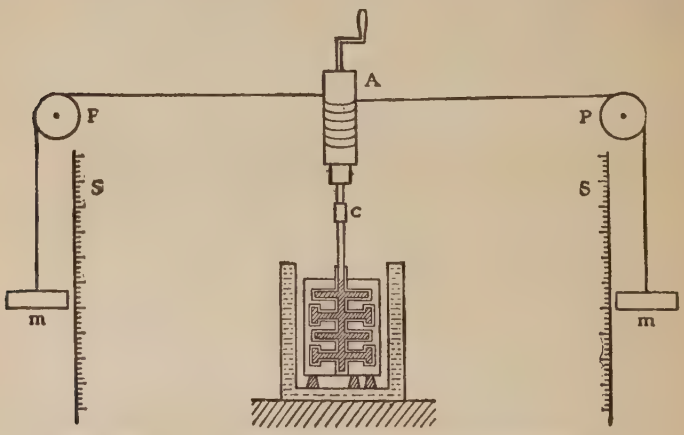

FIG. 106.—Joule's Apparatus for the Determination of J.

paddles, arranged radially on a common vertical axis, and working between four fixed brass vanes as shown in the cross-sectional view in the figure. The necessity for the vanes arises from the fact that the rotation of the paddles would set up a permanent rotation of the water, thus endowing it with kinetic energy instead of heat. The presence of the vanes prevents this rotation, and thus all the energy of rotation of the paddles is entirely available for transference to the water in the form of heat. The motion of the paddles was effected by means of a spindle A, to which they were attached by means of a detachable coupling *c*. The spindle A was rotated by the falling of two equal weights *m*

connected to A by means of thread wound over pulleys P, the distance through which the weights fell being indicated by the vertical scales S. In carrying out the experiment, the height of fall was noted, and the coupling c was detached from the paddles. The weights were then hauled up again by means of the handle of A; the paddles were again attached, and the weights once more allowed to fall. Twenty falls in all were allowed, and the rise in temperature of the water in the calorimeter was carefully noted. All the mountings and parts were carefully designed to give a minimum of friction, but such residual friction as did exist was carefully ascertained and allowed for. Subsidiary experiments were also carried out with a view to determining the corrections necessary for radiation and conduction losses. An additional correction was also made for the kinetic energy of the weights on being suddenly stopped at the end of each fall.

Allowing for these,

Let m_1 = mass of water in calorimeter.
,, c_1 = specific heat of water in calorimeter.
,, W = water equivalent of calorimeter and contents.
,, t_1 = initial temperature of calorimeter and contents.
,, t_2 = final temperature of calorimeter and contents.
,, m = mass of each falling weight.
,, h = height of fall.
,, n = number of falls during the experiment.

$\therefore$ Work done during one fall of weights = force × distance
$= 2mngh$ ergs.

$\therefore$ Work done during n falls of weights $= 2mngh$ ergs.

Also, the heat created by this amount of work

$$= (m_1c_1 + \mathrm{W})(t_2 - t_1) \text{ calories.}$$

Hence $(m_1c_1 + \mathrm{W})(t_2 - t_1)$ calories of heat are equivalent to $2mngh$ ergs of work.

$\therefore$ 1 calorie of heat is equivalent to

$$\frac{2mngh}{(m_1c_1 + \mathrm{W})(t_2 - t_1)} \text{ ergs,}$$

which determines the value for J.

In Joule's actual experiments at Manchester, the British system of units was employed. The first fifty experiments gave the following average readings and results :

Mass of water employed, including water equivalent of calorimeter . .	= 13·926 lb.
Mass of two weights together . .	= 58·022 lb.
Total distance travelled in twenty falls .	= 105·0207 ft.
Rise in temperature of the water (corrected for radiation, etc.) . . .	= 0·5632° F.

∴ Actual work done by falling weights = 58·022 × 105·0207
= 6093·479 ft.-lb.

From this Joule deducted 26·365 ft.-lb. as a correction for residual kinetic energy, etc.

∴ Amount of energy entering the calorimeter = 6093·479 − 26·365
= 6067·114 ft.-lb.

Joule took the specific heat of water as unity.

∴ Heat developed in calorimeter = 13·926 × 1 × 0·5632
= 7·8431 B.T.U.'s.

Hence 7·8431 B.T.U.'s = 6067·114 ft.-lb.

$$\therefore \quad 1 \text{ B.T.U.} = \frac{6067 \cdot 114}{7 \cdot 8431} = 773 \cdot 56 \text{ ft.-lb.}$$

Joule's experiment was repeated in 1877–78, under improved conditions, by *Rowland*, who substituted a steam-engine for the falling weights. The calorimeter was suspended from a wire, and the natural tendency for the wire to twist in consequence of the rotation of the paddles was counteracted by an opposing couple applied tangentially at the top of the calorimeter by two equal weights. The paddle-axis projected downwards through the base instead of upwards, as in Joule's experiment.

Friction-Cones Method.— The usual laboratory experiment for the determination of J is illustrated in Fig. 107. Rigidly fixed in a brass cylinder A by means of an ebonite base *e* and ring *d* is a brass cone B. Fitting smoothly into B is a similar brass cone C containing a known weight of water. A wooden disc G is mounted on C. The outer cone B is made to rotate rapidly by means of a pulley H connected with a small motor,

but the inner cone C is kept still by means of a weight W connected to the disc G by means of a string passing over a pulley P. The speed of H is initially varied until it attains a value such

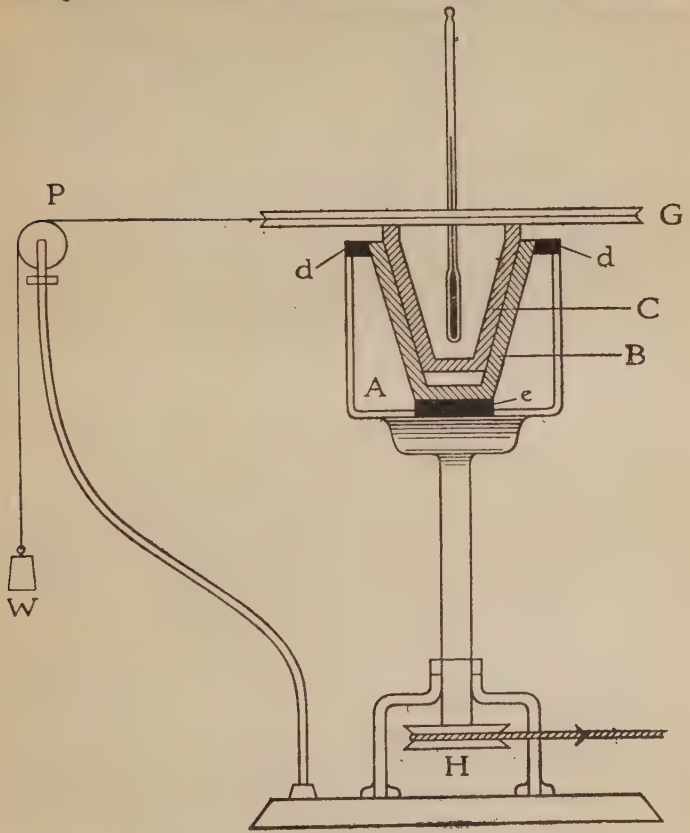

FIG. 107.—Laboratory Method for J.

that W neither moves up nor down. The number of revolutions made by H is indicated by a counter attached to the mechanism. As before, the rise in temperature of the water in the inner cone is noted.

If d be the diameter of the disc G, then the tendency for the inner cone to rotate is counterbalanced by an opposing couple whose forces are Wg and whose arm is a.

Also, from elementary mechanics,

Work done by a couple = moment of couple × angle of rotation.

$\therefore$ work done by a couple in 1 revolution
$= \text{moment of couple} \times 2\pi$
$= Wga \times 2\pi$

$\therefore$ work done by a couple in n revolutions
$= Wga \times 2\pi n$ ergs,

where n = number of revolutions indicated by the counter.

Let m = mass of water in cone C.
" c = its specific heat.
" x = water equivalent of the two cones.
" t_1 = initial temperature of the water.
" t_2 = final temperature of the water.

$\therefore$ heat created $= (mc+x)(t_2-t_1)$ calories.

$\therefore$ $(mc+x)(t_2-t_1)$ calories $= Wga \times 2\pi n$ ergs.

$$\therefore \quad J = \frac{Wga \times 2n}{(mc+x)(t_2-t_1)} \text{ calories.}$$

EXAMPLES ON CHAPTER XV

1. What do you understand by the statement that heat is a form of energy? Describe experiments to illustrate the grounds for this statement.
2. Trace as far back as you can through its various transformations the energy obtained from the water in a waterwheel.
3. 1 pound of coal in burning can raise 8000 pounds of water 1 degree. Used in an engine the coal supplies 1,400,000 foot-pounds of work per pound burnt. What fraction of the heat is transformed to work? (Mech. equiv. = 1400 foot-pounds per pound degree C.)
4. When, how, and where is heat developed as the equivalent of work done when a man (*a*) jumps down on soft ground, (*b*) slides slowly down a rope, (*c*) walks downstairs?
5. An iron ball of mass 1 kilogramme is dropped from a height of 50 metres on to a sheet of ice in which it becomes embedded.

If the ice is at 0° C., and the ball is also at this temperature just previous to impact, explain why some ice will melt, and calculate the amount. ($g = 981$ cms. per second per second, latent heat of fusion of ice = 80 calories, and mechanical equivalent of heat = 4·2 Joules per calorie.)

6. What experiments have led to the enunciation of the principle of the conservation of energy ?
7. A copper calorimeter weighing 100 grams contains 900 grams of water at 15° C. The water is stirred by a paddle which makes 1000 revolutions. The driving couple is 10^8 dyne cms. The temperature of the water rises to 30° C. Calculate the mechanical equivalent of heat.
8. What do you understand by the mechanical equivalent of heat ? A shell has an average specific heat of 0·1. It hits a target with a velocity of 2000 feet per second, and is brought to rest by it. If the heat is all in the shell, find the temperature rise that ensues. ($J = 1390$ foot-pounds per pound degree C.)
9. Define the mechanical equivalent of heat, and explain how it has been determined. 100 grams of water contained in a copper calorimeter weighing 20 grams are stirred with a rotating stirrer. After the stirrer has made 1000 revolutions it is found that the temperature of the water has risen to 5° C. Calculate the couple required to drive the stirrer, given mechanical equivalent of heat $= 4{\cdot}2 \times 10^7$ ergs per gram degree, specific heat of copper 0·1.
10. Some lead shot are allowed to fall 100 times down a tube 80 cms. long. If $J = 4{\cdot}2 \times 10^7$ ergs per calorie, find the rise in temperature of the shot.
11. If 1 pound of petrol contains 18,000 B.T.U.'s of heat, and if a half-pound of petrol supplies 1,980,000 foot-pounds of useful work, what fraction of the total heat is wasted ?
12. The water at Niagara Falls drops 160 feet. How much warmer should the water be at the bottom than at the top if $J = 778$ foot-pounds per B.T.U. ?
13. From what height must a hailstone at 0° C. fall in order just to melt on reaching the ground, assuming that all the energy of the fall is converted into heat without loss ? ($J = 4{\cdot}2 \times 10^7$ ergs per calorie.)
14. To drill a hole in an iron block a power of 0·6 H.P. is applied for $2\frac{1}{2}$ minutes. How much heat is developed ?
15. A leaden bullet strikes a target and is just melted. Assuming that all its energy is converted into heat, and that the striking velocity is 35,000 cms. per second, find the initial temperature of the bullet, given that the specific heat of lead = 0·03, the melting-point = 325° C., the latent heat of fusion = 5·4, and $J = 4{\cdot}2 \times 10^7$ ergs.

16. A bomb is dropped from a stationary airship at 16,000 feet. If half the heat generated by the impact with the ground is absorbed by the bomb, what will be its rise in temperature ? The wind and air resistances may be neglected.
17. A lorry weighing 8 tons is travelling at 35 miles per hour when the brakes are applied to stop the lorry. How much heat is generated thereby ?
18. How has the relation of mechanical work to heat energy been established ? Give a brief historical account of the development of the principle known as the mechanical equivalent of heat, and include a short description of one conclusive experiment. Explain how the relation is employed as a basis for the comparison of the relative merits of two petrol engines, tests upon which showed the following :

Engine.	Duration of Test.	Mean Horse-Power during Test.	Total Petrol Consumed (pounds).	Calorific Value of Petrol in B.T.U. per pound.
A	1·25 hrs.	129	116	18,700
B	1·5 hrs.	138	123	19,200

19. Illustrate the statement that heat is a form of energy by reference to the changes of energy forms met with in the running of a motor-car. State precisely what you understand by the statement that "mechanical equivalent of heat is 778 foot-pounds per B.T.U." If coal costs 33s. per ton and has a heating value of 14,400 B.T.U.'s per pound, and petrol costs 1s. 10d. per gallon and has a heating value of 18,000 B.T.U.'s per pound, compare their heating values for the same money outlay, taking 1 gallon of petrol as weighing 7·3 pound.

APPENDIX

TABLES OF CONSTANTS

TABLE I

COEFFICIENTS OF LINEAR EXPANSION PER DEGREE CENTIGRADE

Aluminium	0·000023	Lead	0·000029
Brass	0·0000188	Oak	0·00005
Copper	0·0000172	Platinum	0·000089
Gold	0·0000144	Quartz	0·0000005
Glass	0·000085	Silver	0·000019
,, (pyrex)	0·000003	Steel	0·000012
Invar (nickel steel)	0·00000087	Tin	0·000022
Iron	0·0000122	Zinc	0·000029

TABLE II

COEFFICIENTS OF CUBICAL EXPANSION PER DEGREE CENTIGRADE

Alcohol	0·00105	Olive oil	0·00074
Benzene	0·00124	Petroleum	0·0010
Ether	0·0015	Turpentine	0·00105
Glycerine	0·00052	Water above 10° C.	0·00043
Mercury	0·000182		

TABLE III

MELTING-POINTS

Hydrogen	−257° C.	Tin	233° C.
Mercury	−39°·5 C.	Bismuth	266° C.
Ice	0° C.	Lead	327° C.
Benzine	7° C.	Zinc	420° C.
Acetic acid	16°·6 C.	Aluminium	657° C.
Tallow	33° C.	Silver	954° C.
Phosphorus	43° C.	Gold	1001° C.
Paraffin wax	43° C.	Copper	1100° C.
Beeswax	65° C.	Cast iron	1200° C.
Napthalene	80° C.	Platinum	1900° C.
Sulphur	114° C.	Iridium	1950° C.

TABLE IV

BOILING-POINTS

Ether	34°·5 C.	Xylene	140° C.
Carbon bisulphide	47° C.	Turpentine	157° C.
Chloroform	61° C.	Nitrobenzine	210° C.
Ethyl alcohol	78°·1 C.	Paraffin oil	280° C.
Benzene	81° C.	Glycerine	290° C.
Water	100° C.	Linseed oil	315° C.
Toluene	111° C.	Mercury	357° C.
Amyl alcohol	130° C.	Sulphur	445° C.

TABLE V

DATA FOR USE WITH THE WET AND DRY BULB THERMOMETERS

Dry Bulb Reading.	Difference between Wet and Dry Bulb Readings.									
	0° C.	1° C.	2° C.	3° C.	4° C.	5° C.	6° C.	7° C.	8° C.	9° C.
0° C.	4·6	3·7	2·9	2·1	1·3	..	..	..	..	..
1° C.	4·9	4·0	3·2	2·4	1·6	0·8	..	..	..	..
2° C.	5·3	4·4	3·4	2·7	1·9	1·0	..	..	..	..
3° C.	5·7	4·7	3·7	2·8	2·2	1·3	..	..	..	..
4° C.	6·1	5·1	4·1	3·2	2·4	1·6	0·8	..	..	..
5° C.	6·5	5·5	4·5	3·5	2·6	1·8	1·0	..	..	..
6° C.	7·0	5·9	4·9	3·9	2·9	2·0	1·1	..	..	..
7° C.	7·5	6·4	5·3	4·3	3·3	2·3	1·4	0·4	..	..
8° C.	8·0	6·9	5·8	4·7	3·7	2·7	1·7	0·8	..	..
9° C.	8·6	7·4	6·3	5·2	4·1	3·1	2·1	1·1	0·2	..
10° C.	9·2	8·0	6·8	5·7	4·6	3·5	2·5	1·5	0·5	..
11° C.	9·8	8·6	7·4	6·2	5·1	4·0	2·9	1·9	0·9	..
12° C.	10·5	9·2	8·0	6·8	5·6	4·5	3·4	2·3	1·3	..
13° C.	11·2	9·8	8·6	7·3	6·2	5·0	3·9	2·8	1·7	..
14° C.	11·9	10·6	9·2	8·0	6·7	5·6	4·4	3·3	2·2	1·1
15° C.	12·7	11·3	9·9	8·6	7·4	6·1	5·0	3·8	2·7	1·6
16° C.	13·5	12·1	10·7	9·3	8·0	6·8	5·5	4·3	3·2	2·1
17° C.	14·4	13·0	11·5	10·1	8·7	7·4	6·2	4·9	3·7	2·6
18° C.	15·4	13·8	12·3	10·9	9·5	8·1	6·8	5·5	4·3	3·1
19° C.	16·4	14·7	13·2	11·7	10·3	8·9	7·5	6·2	4·9	3·7
20° C.	17·4	15·7	14·1	12·6	11·1	9·7	8·3	6·9	5·6	4·3
21° C.	18·5	16·8	15·1	13·5	12·0	10·5	9·0	7·6	6·3	5·0
22° C.	19·7	17·9	16·2	14·5	12·9	11·4	9·9	8·4	7·0	5·7
23° C.	20·9	19·0	17·3	15·6	13·9	12·3	10·8	9·2	7·8	6·4
24° C.	22·2	20·3	18·4	16·6	14·9	13·3	11·7	10·1	8·7	7·2

TABLE VI

Pressure of Water Vapour in Millimetres of Mercury

Temperature.	Pressure.	Temperature.	Pressure.	Temperature.	Pressure.	Temperature.	Pressure.	Temperature.	Pressure.
° C.		° C.		° C.		° C.		° C.	
0·0	4·5687	7·0	7·4660	14·0	11·8835	21·0	18·4659	28·0	28·0654
0·2	4·6350	7·2	7·5685	14·2	12·0380	21·2	18·6937	28·2	28·3942
0·4	4·7022	7·4	7·6722	14·4	12·1943	21·4	18·9240	28·4	28·7265
0·6	4·7703	7·6	7·7772	14·6	12·3523	21·6	19·1567	28·6	29·0622
0·8	4·8393	7·8	7·8834	14·8	12·5122	21·8	19·3920	28·8	29·4013
1·0	4·9091	8·0	7·9909	15·0	12·6739	22·0	19·6297	29·0	29·7439
0·2	4·9798	8·2	8·0998	15·2	12·8374	22·2	19·8701	29·2	30·0900
0·4	5·0515	8·4	8·2099	15·4	13·0027	22·4	20·1130	29·4	30·4396
0·6	5·1240	8·6	8·3214	15·6	13·1700	22·6	20·3586	29·6	30·7928
0·8	5·1975	8·8	8·4342	15·8	13·3392	22·8	20·6068	29·8	31·1494
2·0	5·2719	9·0	8·5484	16·0	13·5103	23·0	20·8576	30·0	31·5096
2·2	5·3472	9·2	8·6641	16·2	13·6832	23·2	21·1110	..	..
2·4	5·4235	9·4	8·7810	16·4	13·8582	23·4	21·3672	..	..
2·6	5·5008	9·6	8·8993	16·6	14·0051	23·6	21·6262	..	..
2·8	5·5790	9·8	9·0189	16·8	14·2141	23·8	21·8879	..	..
3·0	5·6582	10·0	9·1398	17·0	14·3950	24·0	22·1524	40·0	54·9062
3·2	5·7383	10·2	9·2623	17·2	14·5779	24·2	22·4196	50·0	91·9826
3·4	5·8195	10·4	9·3863	17·4	14·7630	24·4	22·6898	60·0	148·7919
3·6	5·9017	10·6	9·5117	17·6	14·9501	24·6	22·9628	70·0	233·1000
3·8	5·9850	10·8	9·6387	17·8	15·1392	24·8	23·2386	80·0	354·2800
4·0	6·0693	11·0	9·7671	18·0	15·3304	25·0	23·5174	90·0	525·4500
4·2	6·1546	11·2	9·8969	18·2	15·5237	25·2	23·7991	100·0	760·0000
4·4	6·2410	11·4	10·0283	18·4	15·7192	25·4	24·0838	..	..
4·6	6·3285	11·6	10·1614	18·6	15·9169	25·6	24·3714	..	..
4·8	6·4171	11·8	10·2960	18·8	16·1168	25·8	24·6620	..	..
5·0	6·5067	12·0	10·4322	19·0	16·3189	26·0	24·9556	..	..
5·2	6·5974	12·2	10·5700	19·2	16·5233	26·2	25·2523	..	..
5·4	6·6893	12·4	10·7093	19·4	16·7299	26·4	25·5521	..	..
5·6	6·7824	12·6	10·8503	19·6	16·9388	26·6	25·8551	..	..
5·8	6·8765	12·5	10·9928	19·8	17·1499	26·8	26·1612	..	..
6·0	6·9718	13·0	11·1370	20·0	17·3632	27·0	26·4705	..	..
6·2	7·0682	13·2	11·2829	20·2	17·5789	27·2	26·7830	..	..
6·4	7·1658	13·4	11·4304	20·4	17·7971	27·4	27·0987	..	..
6·6	7·2646	13·6	11·5797	20·6	18·0176	27·6	27·4177	..	..
6·8	7·3647	13·8	11·7307	20·8	18·2406	27·8	27·7399	..	..

TABLE VII

SPECIFIC HEATS

I. Solids

Aluminium	0·212	Lead	0·031
Bismuth	0·030	Limestones	0·216
Brass	0·094	Nickel	0·109
Brick	0·195	Plaster	0·200
Charcoal	0·242	Platinum	0·032
Coal	0·204	Silver	0·056
Copper	0·093	Steel	0·120
Earth	0·200	Stone (average)	0·210
Glass	0·194	Tin	0·056
Gold	0·032	Wood (birch)	0·480
Human body	0·830	,, (oak)	0·570
Ice	0·500	,, (pine)	0·467
Iron (wrought)	0·113	Zinc	0·093
,, (cast)	0·130		

II. Liquids

Acetic acid	0·46	Ether	0·529
Alcohol (ethyl)	0·615	Glycerine	0·550
,, (methyl)	0·590	Kerosine	0·683
Ammonium (liquid)	1·012	Mercury	0·033
Benzine	0·450	Petroleum	0·460 to 0·510
Benzol	0·416	Sulphuric acid	0·336
Carbon disulphide	0·240	Turpentine	0·459
Chloroform	0·235	Water	1·000

III. Gases (at Constant Pressure)

Air	0·238	Hydrogen	3·410
Ammonia (gas)	0·508	Nitrogen	0·244
Carbon dioxide	0·247	Oxygen	0·217
Carbon monoxide	0·242	Steam	0·488

IV. Foods

Apples	0·910	Milk	0·940
Beef (lean)	0·770	Olive oil	0·309
Celery	0·960	Potato	0·800
Cream (light)	0·950	Salt	0·170
Eggs	0·790	Veal	0·700
Lard (melted)	0·510		

TABLE VIII

LATENT HEATS

Latent Heat of Fusion (Calories)

Mercury	2·8	Tin	14·6
Ice	80·0	Bismuth	12·6
Nitrobenzene	22·3	Cadmium	13·7
Formic acid	55·6	Lead	5·7
Acetic acid	43·0	Zinc	28·0
Yellow phosphorus	5·2	Silver	21·1
Spermaceti	37·0	Cast iron (white)	33·0
Beeswax (unbleached)	42·3	,, (grey)	23·0
Napthalene	35·6	Platinum	27·2
Sulphur	9·4		

Latent Heat of Vaporisation (Calories)

Carbon dioxide	49·32	Benzol	93·0
Anhydrous ether	90·7	Absolute alcohol	208·9
Carbon disulphide	86·67	Water	537·0
Acetone	130·0	Acetic acid	102·0
Chloroform	80·0	Turpentine	74·0
Bromine	45·6	Mercury	62·0
Methyl alcohol	262·86	Sulphur	362·0

TABLE IX

THERMAL CONDUCTIVITIES

I. Relative Thermal Conductivities

(In terms of the best conducting substance, silver.)

Silver	1·0	Hard wood	0·0006
Copper	0·9	Linoleum	0·00036
Aluminium	0·5	Dry soil	0·00034
Brass	0·27	Asbestos	0·00025
Iron	0·14	Flannel	0·00023
Platinum	0·06	Silk	0·00022
Rock (average)	0·006	Sawdust	0·00018
Calico	0·0053	Soft wood	0·00075
Brick (average)	0·005	Paper	0·00013
Porcelain	0·0025	Cork	0·00012
Glass	0·0016	Wool	0·00010
Water	0·0014	Hair felt	0·00010
Sand	0·0009	Cotton wool	0·00009
Feathers	0·000057	Dry air	0·00005

II. Absolute Thermal Conductivities at Normal Temperature (C.G.S. Units)

(*a*) Metals

Silver	1·09	Iron	0·17
Copper	1·04	Tin	0·15
Magnesium	0·37	Lead	0·084
Aluminium	0·344	Antimony	0·04
Zinc	0·28	Bismuth	0·02

(*b*) Non-Metallic Solids.

Quartz (parallel to principal axis)	0·030	Cork	0·0004
Quartz (perpendicular to principal axis)	0·016	Fire felt	0·00021
Ice	0·0056	Asbestos paper	0·00017
Marble	0·005	Wool felt	0·000125
Glass	0·002	Calico	0·00014
Ebonite	0·0004	Cork board	0·0001
Wood	0·0003 to 0·0004	Cotton wool (loose)	0·0001
		Hair felt	0·000085
		Pure wool	0·000085 to 0·00009

(*c*) Liquids

Absolute alcohol	0·000423	Turpentine	0·000260
Chloroform	0·000288	Water	0·00136
Ether	0·000303	Mercury	0·2
Glycerine	0·000670		

(*d*) Gases

Dry air	0·0000516	Hydrogen	0·00033
Carbon dioxide	0·0000273		

ANSWERS TO QUESTIONS

CHAPTER I

1. (*a*) 55°·4 F., 113° F., 244°·4 F. ; (*b*) −9°·44 C., 6°·11 C., 34°·4 C.
2. −40, 10, 1·053 : 1.
14. −430·6, −39·1, 32, 24·6, 61·88, 91·4, 109·4, 129·2, 149·0, 176·0, 237·2, 451·4, 510·8, 620·6, 748·0, 1214·6, 1749·2, 1833·8, 2012·0, 2192·0, 3452·0, 3542·0.
15. 94·1, 116·6, 141·8, 172·6, 177·8, 212, 231·8, 266·0, 284·0, 314·6, 410·0, 536·0, 554·0, 599·0, 674·6, 833·0.

CHAPTER II

1. 23°·53 C. **3.** 0·0575 foot. **4.** 0·0396 inch. **5.** 45·035 cms.
6. 0·0000121. **7.** (*a*) 313·55 sq. cms. ; (*b*) 314·03 sq. cms.
10. 10,008·25 cubic yards. **12.** 0·0045 c.c. per degree.
13. 0·000045 per cent. **14.** 454°·5 C. **15.** 1·09376 yards.
16. 334°·56 C. **17.** 0.0036 inch.
18. (*a*) 0·0097 inch ; (*b*) 0·8007 cubic inch.
19. 3·00216 inches. **20.** 17°·36 C. **21.** 0·0000312.
22. 0·0297 inch. **23.** 888⅜° C. **24.** 0·0012 inch.
25. 2·9933 inches. **26.** 0·00000476.

CHAPTER III

2. 0·00058 **4.** 100·54. **10.** 117°·7C. **14.** 90°·68 C.
16. 291·2 gals. **17.** 0·0001802 **18.** 77·16 **19.** 18·621 c.c.

CHAPTER IV

2. 1343°·6 C. **4.** 85/373. **7.** 30 inches. **8.** 0·1819 cubic foot.
9. 172·46 litres. **11.** 1·2 feet. **12.** 150 cylinders.
13. 947°·8 C. **16.** 0·00201 gram per c.c.
17. (*a*) 91·1 lb. per square inch ; (*b*) 86·8 lb. per square inch.
18. 248·8 lb. **19.** 101·44. **20.** 415°·6 C.
21. 15°·16 C. **22.** 0·00162. **23.** 60·01 cubic cm.

CHAPTER VIII

4. 65·5 cms. **14.** 89·23 c.c. **15.** 78·45 cubic inches.
16. 687·57 cubic inches. **19.** (*a*) 46·65 mm. ; (*b*) 66°·5 C.

CHAPTER IX

3. 0·012. **6.** 82·7 per cent. **7.** 3·40 c.c. **8.** 2 : 1.
11. 0·4 nearly. **12.** 86·2 per cent. **13.** 0·001206 gram per c.c.
14. 11°·9 C. ; 69·5 per cent. **15.** $8{\cdot}51 \times 10^{-6}$ grams per c.c.

CHAPTER X

5. 0°·98 C. **8.** 31°·7 C. **9.** 0·638. **12.** $\frac{3}{4}$ lb. iron ; $1\frac{1}{4}$ lb. copper.
13. W.E., 12 ; specific heat, 0·112. **16.** 4·76 grams.
17. 2·71 cubic feet. **18.** Hot, $10\frac{5}{13}$ gals. ; cold, $19\frac{5}{13}$ gals. **19.** 0·437.
20. 1049°·2 F. **21.** 0°·386 F. per second. **22.** 955°·4 C.
23. 90,000 B.T.U.'s.

CHAPTER XI

2. 787·5 calories. **3.** 635. **4.** 19 grams. **5.** 8·34 grams.
6. 7·0625 grams. **7.** 2448 calories per minute.
8. 2416·6 tons ; 5345·4 tons. **9.** 11·12 grams.
12. 10 hours 36 minutes. **13.** 530·4 calories. **14.** 0·5.
15. 9·00 cms. **16.** 0·0976. **18.** 543·7 calories.
19. 2566 calories. **20.** 2,288,000 calories. **21.** 1120 B.T.U.'s.
22. 15·95 grams. **25.** 569·7 calories. **26.** 0·1018.
27. 6·11 grams. **28.** 0·09935. **29.** 144 ; 966·6.

CHAPTER XIII

6. 750,000 calories. **7.** 1,800,000 calories. **8.** 0·0003.
9. 53·3 minutes if density = 1. **13.** 0·000177. **14.** 4·2 : 1.
15. $30{\cdot}5 \times 10^{6}$ calories. **16.** $5{\cdot}44 \times 10^{5}$ calories. **18.** 403·2 B.T.U.'s.
19. 3667·52 B.T.U.'s.

CHAPTER XV

4. $\frac{1}{8}$. **6.** 1·46 grams. **8.** 14,388°·5 C. **9.** 341,083 units.
10. 6° C. **11.** 0·717. **12.** 6°·58 F. **13.** 342·6 kilometres.
14. 63·6 B.T.U.'s **15.** 18°·3 C. **16.** 1406° C.
17. 68,300 B.T.U.'s. **19.** 13·6 : 1.

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